Dynamics and time series: theory and applications

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Lecture 8, Mar 2, 2009

- Lecture 1: An introduction to dynamical systems and to time series. Periodic and quasiperiodic motions. (Tue Jan 13, 2 pm - 4 pm Aula Bianchi)
- Lecture 2: Ergodicity. Uniform distribution of orbits. Return times. Kac inequality Mixing (Thu Jan 15, 2 pm - 4 pm Aula Dini)
- Lecture 3: Kolmogorov-Sinai entropy. Randomness and deterministic chaos. (Tue Jan 27, 2 pm - 4 pm Aula Bianchi)
- Lecture 4: Time series analysis and embedology. (Thu Jan 29, 2 pm - 4 pm Dini)
- Lecture 5: Fractals and multifractals. (Thu Feb 12, 2 pm - 4 pm Dini)
- Lecture 6: The rhythms of life. (Tue Feb 17, 2 pm - 4 pm Bianchi)
- Lecture 7: Financial time series. (Thu Feb 19, 2 pm - 4 pm Dini)
- Lecture 8: The efficient markets hypothesis. (Tue Mar 3, 2 pm - 4 pm Bianchi)
- Lecture 9: A random walk down Wall Street. (Thu Mar 19, 2 pm - 4 pm Dini)
- Lecture 10: TBA. (Tue Mar 24, **11 am– 1 pm** aula ?? Rosario Mantegna)

- Seminar I: Waiting times, recurrence times ergodicity and quasiperiodic dynamics (D.H. Kim, Suwon, Korea; Thu Jan 22, 2 pm - 4 pm Aula Dini)
- Seminar II: Symbolization of dynamics. Recurrence rates and entropy (S. Galatolo, Università di Pisa; Tue Feb 10, 2 pm - 4 pm Aula Bianchi)
- Seminar III: Heart Rate Variability: a statistical physics point of view (A. Facchini, Università di Siena; Tue Feb 24, 2 pm - 4 pm Aula Bianchi)
- Seminar IV: Study of a population model: the Yoccoz-Birkeland model (D. Papini, Università di Siena; Thu Feb 26, 2 pm - 4 pm Aula Dini)
- Seminar V: Scaling laws in economics (G. Bottazzi, Scuola Superiore Sant'Anna Pisa; Tue Mar 17, 2 pm - 4 pm Aula Bianchi)
- Seminar VI: Complexity, sequence distance and heart rate variability (M. Degli Esposti, Università di Bologna; Thu Mar 26, 2 pm - 4 pm Aula Dini)
- Seminar VII: Forecasting (M. Lippi, Università di Roma; late april, TBA)

Today's bibliography:

S.J. Taylor "Asset price dynamics, volatility and prediction" Princeton University Press, Ch. 5

K. Cuthbertson "Quantitative Financial Economics" Chapter 4 Valuation and 5 The efficient markets hypothesis

Eugene Fama: "Foundations of Finance" Chapter 5: Efficient Capital Markets

Elroy Dimson and Massoud Mussavian: "A brief history of market efficiency " European Financial Management, Volume 4, Number 1, March 1998, pp 91-193

M. Yor (Editor): Aspects of Mathematical Finance, Springer 2008, especially chapters 1 "Financial Uncertainty, Risk Measures and Robust Preferences" by H. Föllmer and chapter 2 "The Notion of Arbitrage and Free Lunch in Mathematical Finance" by W. Schachermayer

Suggested web lectures:

1. Robert Shiller (Yale): Efficient markets vs. excess volatility (Lecture 6 of 26 from the course Financial Markets)
http://academicearth.org/lectures/efficient-markets-excess-volatlity

Several theories in finance relate to stock price analysis and prediction. The efficient markets hypothesis states that stock prices for publicly-traded companies reflect all available information. Prices adjust to new information instantaneously, so it is impossible to "beat the market." Furthermore, the random walk theory asserts that changes in stock prices arise only from unanticipated new information, and so it is impossible to predict the direction of stock prices. Using statistical tools, we can attempt to test the hypotheses and to predict future stock prices. These tests show that efficient markets theory is a half-truth: it is difficult but not impossible for some people to beat the market.

2. http://academicearth.org/lectures/andrew-redleaf

Andrew Redleaf, a Yale graduate and manager of Whitebox Advisors, a hedge fund, discusses his experience with financial markets. He addresses one of the fundamental questions in finance--whether or not markets are efficient--and concludes that although they don't seem to be efficient, beating the market is very difficult. Mr. Redleaf discusses his thoughts about psychological barriers that make markets inefficient. He also comments on his beliefs regarding risk management and how people are compensated for mitigating risks, rather than for taking on risk as is often perceived. He ends by answering several questions from students.

Fundamentals of investing

Investment returns are strongly related to their risk level
Usually and loosely risk is quantified using volatility (standard deviation)
U.S. Treasury bills /bonds (short/long term bonds 1month-1year / 2-30 years): very safe (until now…) and very low/medium yield. Most of the price uncertainty for longer term bonds comes from the effect of inflation
T.I.P. : inflation indexed bonds which guarantee a positive real return
Stocks: risky but higher returns (on the long run…). Companies sell shares of stock to raise capital: they ``go public'' by agreeing to sell a certain number of shares on an exchange. Each share represents a given fraction of the ownership of the company.
Certain stocks pay *dividends*, cash payments reflecting profits returned to shareholders. Other stocks reinvest all returns back into the business.
In principle, what people will pay for a stock reflects the health of its current business, future prospects, and expected returns. But the current *price* of a stock is completely determined by what people are willing to pay for it. If there were no differences of opinion as to the value of a stock, there would be no trading.

Stock prices and indices

Stock indices are typically weighted averages of the prices of the component stocks. Usually the weights are proportional to the market capitalization= (price of a share)*(number of existing shares) of the stock.

The same formulae as before are used to calculate returns from index levels. Very often dividends are excluded from the index.

Dow Jones Industrial Average: 30 U.S. stocks (corresponding to 30 leading companies), price weighted

S&P500: 500 U.S. stocks, capitalization weighted

Stoxx 600: 600 european stocks, capitalization weighted

Financial derivatives

These are products whose value depends on share prices. The value of trade of financial derivatives far outweighs the direct trade in shares.

Call option: right to buy for pre-set strike price on a specific date (European) or by a specific date (American).

Put option: right to sell for fixed strike price.

Why buy an option?

1. Hedge against risk: Suppose you think a share you hold is going to fall in value and you decide to sell now and buy back later. Buying a call option will let you buy back at an established price if your guess is wrong.

2. Speculation: You think a share priced 100$ is going to go up. If you are right and it goes up to 110$ you gain 10%. Suppose instead you can buy a call option with a strike price of 100$ for 5$ a share. If they go up to 110$ you can exercise the option and make 10$ a share - gain is now 100%.

Downside - if the share price has fallen by the expiry date your investment is worth nothing.

Long and short positions

Long position - when you stand to gain if the asset price rises, obvious case being if you own it.
Short position - you gain if the price goes down, for example if you have borrowed a share and sold it, then need to repurchase it to return the loan. This is called selling short.

Risk-free assets

Another crucial element of the theory is the existence of risk-free assets which define a risk free interest rate. Over the short term these could be bank accounts where the rate of interest for corporate customers is essentially set by the Bank of England base rate (in the UK). For longer periods the risk-free interest rate is set by the yield on government bonds which pay a fixed rate of interest until maturity then their face value (usually £1) at maturity.
Any risk free asset must carry the same rate of interest over a given period. Otherwise an institution could take a long position on the one giving the higher rate and a short position in the other, yielding an arbitrage opportunity. Buying and selling assets involves dealing costs and commission. In all that we do we neglect these and look at the basic price before they are added.

Arbitrage and market efficiency

Arbitrage is a trading strategy which takes advantage of two or more securities being inconsistently priced relative to each other. An arbitrage opportunity is the possibility to make a profit in a financial market without risk and without net investment of capital. An arbitrage is a transaction that involves no negative cash flow at any probabilistic or temporal state and a positive cash flow in at least one state; in simple terms, a risk-free profit.

The principle of no arbitrage states that a mathematical model of a financial market should not allow for arbitrage possibilities.

This principle is fundamental and can be used for calculating the price of financial assets.

To apply this principle to less trivial cases, we consider a – still extremely simple – mathematical model of a financial market: there are two assets, called the bond and the stock. The bond is riskless, hence – by definition – we know what it is worth tomorrow. For (mainly notational) simplicity we neglect interest rates and assume that the price of a bond equals €1 today as well as tomorrow, i.e.

$$B_0 = B_1 = 1. \tag{1}$$

The more interesting feature of the model is the stock which is risky: we know its value today, say $S_0 = 1$, but we do not know its value tomorrow. We model this uncertainty stochastically by defining S_1 to be a random variable depending on the random element $\omega \in \Omega$. To keep things as simple as possible, we let Ω consist of two elements only, g for 'good' and b for 'bad', with probability $\mathbf{P}[g] = \mathbf{P}[b] = \frac{1}{2}$. We define $S_1(\omega)$ to equal 2 or $\frac{1}{2}$ according to whether

$$S_1(\omega) = \begin{cases} 2 & \text{for } \omega = g \\ \frac{1}{2} & \text{for } \omega = b. \end{cases} \tag{2}$$

Now we introduce a third financial instrument in our model, an *option on the stock* with strike price K: the buyer of the option has the right – but not the obligation – to buy one stock at time $t = 1$ at the predefined price K. To fix ideas let $K = 1$. A moment's reflexion reveals that the price C_1 of the option at time $t = 1$ (where C stands for *contingent* claim) equals

$$C_1 = (S_1 - K)_+, \tag{3}$$

i.e. in our simple example

$$C_1(\omega) = \begin{cases} 1 & \text{for } \omega = g \\ 0 & \text{for } \omega = b. \end{cases} \tag{4}$$

Hence we know the value of the option at time $t = 1$, *contingent on the value of the stock*. But what is the price of the option today?

At this stage the reader might consult the financial section of a newspaper or the web to see some 'life' examples on quoted option prices.

The classical approach, used by actuaries for centuries, is to price contingent claims by taking expectations, which leads to the value $C_0 := \mathbf{E}[C_1] = \frac{1}{2}$ in our example. Although this simple approach is very successful in many actuarial applications, it is not at all satisfactory in the present context. Indeed, the rationale behind taking the expected value as the price of a contingent claim is the following: in the long run the buyer of an option will neither gain nor lose on average. We rephrase this fact in a financial lingo: the performance of an investment in the option would on average equal the performance of the bond. However, a basic feature of finance is that an investment into a risky asset should, on average, yield a better performance than an investment in the bond (for the skeptical reader: at the least these two values should not necessarily coincide). In our 'toy example' we have chosen the numbers such that $\mathbf{E}[S_1] = 1.25 > 1 = \mathbf{E}[B_1]$, so that on average the stock performs better than the bond.

3 Pricing by No Arbitrage

The Notion of Arbitrage and Free Lunch in Mathematical Finance W. Schachermayer

A different approach to the pricing of the option goes like this: we can buy at time $t = 0$ a *portfolio* consisting of $\frac{2}{3}$ of stock and $-\frac{1}{3}$ of bond. The reader might be puzzled about the negative sign: investing a negative amount in a bond – 'going short' in financial lingo – means to borrow money.

One verifies that the value Π_1 of the portfolio at time $t = 1$ equals 1 or 0 depending on whether ω equals g or b. The portfolio 'replicates' the option, i.e.

$$C_1 \equiv \Pi_1. \tag{5}$$

We are confident that the reader now sees why we have chosen the above weights $\frac{2}{3}$ and $-\frac{1}{3}$: the mathematical complexity of determining these weights such that (5) holds true amounts to solving two linear equations in two variables.

The portfolio Π has a well-defined price at time $t = 0$, namely $\Pi_0 = \frac{2}{3}S_0 - \frac{1}{3}B_0 = \frac{1}{3}$. Now comes the 'pricing by no arbitrage' argument: equality (5) implies that we also must have

$$C_0 = \Pi_0 \tag{6}$$

whence $C_0 = \frac{1}{3}$. Indeed, suppose that (6) does not hold true; to fix ideas, suppose we have $C_0 = \frac{1}{2}$ as above. This would allow an arbitrage by buying ('going long in') the portfolio Π and simultaneously selling ('going short in') the option C. The difference $C_0 - \Pi_0 = \frac{1}{6}$ remains as arbitrage profit at time $t = 0$, while at time $t = 1$ the two positions cancel out *independently of whether the random element ω equals g or b.*

What is an efficent capital market?

An efficient capital market is a market which is efficient in processing information: the prices of securities observed at any time are based on “correct” evaluation of all information available at that time. Prices “fully reflect” available information. The prices are always “fair”, they are good indicators of value

The concept of market efficiency had been anticipated at the beginning of the century: Bachelier (1900) writes "past, present and even discounted future events are reflected in market price, but often show no apparent relation to price changes". And also "if the market, in effect, does not predict its fluctuations, it does assess them as being more or less likely, and this likelihood can be evaluated mathematically".

Weak vs. strong efficiency

More formally: a capital market is said to be efficient if it fully and correctly reflects all relevant information in determining security prices. Formally, the market is said to be efficient with respect to some information set, Θ_t , if security prices would be unaffected by revealing that information to all participants. Moreover, efficiency with respect to an information set, Θ_t, implies that it is impossible to make economic profits by trading on the basis of Θ_t.

The weak form of the efficient market hypothesis claims that prices fully reflect the information implicit in the sequence of **past prices**. The semi-strong form of the hypothesis asserts that prices reflect all relevant information that is **publicly** available, while the strong form of market efficiency asserts information that is known to **any** participant is reflected in market prices.

•LE MONDE | 07.01.03 | 12h43

•MIS A JOUR LE 08.01.03 | 13h14

Point de vue

Le mythe funeste des marchés efficients, par Jean-Philippe Bouchaud

Une fois de plus, le repli désordonné des Bourses mondiales et ses conséquences préoccupantes sur l'économie dite *"réelle"* (débâcle financière de géants industriels, baisse auto-entretenue de la confiance, etc.) pose de nombreuses questions sur la nature, le fonctionnement et le rôle, économique et social, des marchés financiers. L'attribution récente – simple coïncidence ou message opportun ? – du prix Nobel d'économie à Daniel Kahnemann et Vernon Smith, deux trouble-fête qui mettent en doute le paradigme dominant, donne une actualité supplémentaire à des questions parfois théoriques, mais qui méritent, me semble-t-il, une plus grande publicité.

L'économie néoclassique présente le marché financier comme un baromètre infaillible de l'activité humaine, dont le rôle cardinal est de donner une valeur précise à tout ce qui peut s'échanger. Le prix de marché est censé représenter la résultante collective des anticipations d'agents parfaitement rationnels. Cette synthèse optimale de l'information, supérieure en qualité à toutes ses parties, permettrait au marché de valoriser correctement et à tout instant un futur économique incertain et de fixer des prix de référence stables qui déterminent nombre de décisions stratégiques. Les prix rationnels devraient, en principe, peu changer au cours du temps, sauf en cas d'informations nouvelles dont l'incidence est majeure.

La théorie des marchés *"efficients"*, parfaitement cohérente du point de vue logique, mais –, on s'en doute – fort peu réaliste, est pourtant le socle théorique de l'ultralibéralisme. Elle est aussi l'une des pierres angulaires de la science économique telle qu'elle est enseignée, depuis plusieurs dizaines d'années, à ceux qui ont et auront la charge de gérer, à des degrés divers, l'économie mondiale. Or les idées recues laissent toujours des traces, surtout lorsqu'elles font partie d'un système intellectuellement séduisant. Dans ce cadre, le marché a toujours raison, puisqu'il se place à un méta-niveau, démiurge inaccessible aux individus. Défier le marché est donc au mieux présompteux, au pis suicidaire.

Cette idée est à la fois confortable et dangereuse. Confortable, comme tout instrument de mesure fiable : le prix du marché tout à la fois guide et justifie les décisions économiques et industrielles. Investir dans les pays émergents dans l'économie Internet ? Oui, puisque les marchés, par les valorisations astronomiques des sociétés *dot-com*, nous disent que cette nouvelle économie atteindra des niveaux de rentabilité inédits. Racheter Orange ou d'autres sociétés technologiques à des prix faramineux qui maintenant grèvent de manière inquiétante les comptes de France Télécom ou de Vivendi ? C'est le prix du marché, indiscutable et tyrannique - qui pourrait en effet se prévaloir de façon crédible d'une clairvoyance supérieure à celle distillée par le consensus collectif ?

More critiques to the EMH

Grossman and Stiglitz ("On the Impossibility of Informatioally Efficient Markets, American Economic Review, 70, 393-408, 1980) argue that perfectly informationally efficient markets are an impossibility. Roughly speaking the idea is more or less that if markets were perfectly efficient, there would be no profit to gathering information, in which case (in an equilibrium world) there would be little reason to trade and markets would eventually collapse.

Alternatively, the degree of market inefficiency determines the effort investors are willing to expend to gather and trade on information, hence a non-degenerate market equilibrium will arise only when there are sufficient profit opportunities, i.e., inefficiencies, to compensate investors for the costs of trading and information-gathering. The profits earned by these attentive investors may be viewed as "economic rents" that accrue to those willing to engage in such activities. Who are the providers of these rents? Black (1986) gave us a provocative answer: "noise traders", individuals who trade on what they consider to be information but which is, in fact, merely noise.

(From A. Lo, The Adaptive Market Hypothesis, Journal of Portfolio Management 2004)

Can Predicable Patterns in Market Returns be Exploited Using Real Money? *Not likely.*

On the occasion of the 30th anniversary of this Journal, which has done so much to enhance the professionalism and the intellectual content of portfolio management, it seems appropriate to examine the shift of belief in the academy as to the efficiency of financial markets and the degree to which stock prices are predictable. In this article, I review some important academic contributions of the past three decades, and present some new empirical evidence concerning stock market predictability.

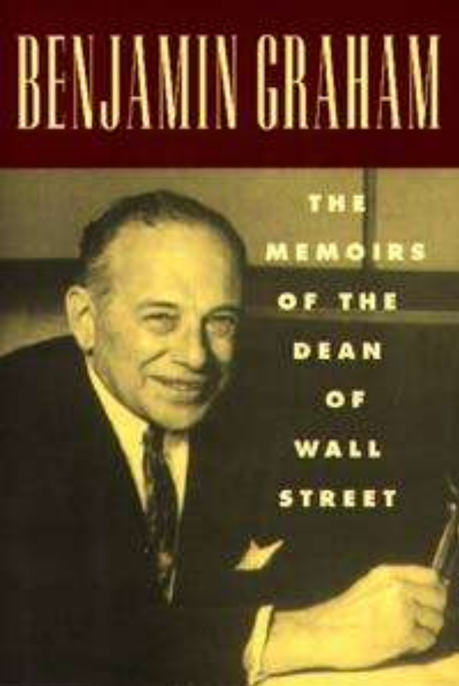
I argue that it is not realistically possible to earn the excess returns that the academic findings about predictability would seem to suggest. I conclude that considerable skepticism is warranted with respect to active portfolio management strategies as well as strategies designed to alter asset allocations over time on the basis of relative valuations.

Burton G. Malkiel is Chemical Bank Chairman's Professor of Economics at Princeton University in Princeton, NJ.

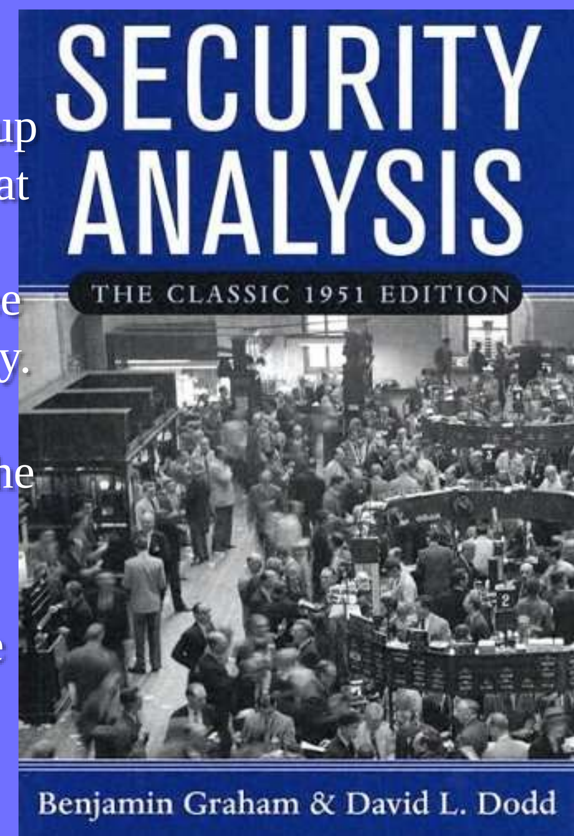
FROM EFFICIENT MARKETS TO STOCK MARKET PREDICTABILITY

In 1970, when Eugene Fama published his survey, "Efficient Capital Markets: A Review of Theory and Empirical Work," there was broad consensus in the academy that our security markets were extremely efficient. Stock prices were assumed to reflect information without delay. If prices accurately represent the future prospects of each firm, then one stock would be just as attractive as another, and an investor who tries to switch from security to security in an attempt to gain excess returns would be unsuccessful.

The notion of efficiency was associated with the view that stock prices would move unpredictably. If prices change only with the receipt of new information, and since true



Benjamin Graham (5/8/1894-9/21/1976)
was an American economist and professional investor. First proponent of value investing, an investment approach he began teaching at Columbia Business School in 1928 and subsequently refined with David Dodd through various editions of their famous book *Security Analysis*. His most famous disciples is Warren Buffet, who credits Graham as grounding him with a sound intellectual investment framework. Graham recommended that investors spend time and effort to analyze the financial state of companies. When a company is available on the market at a price which is at a discount to its fair value, a *margin of safety* exists, which makes it suitable for investment.



Graham's favorite allegory is that of Mr. Market, a fellow who turns up every day at the stock holder's door offering to buy or sell his shares at a different price. Often, the price quoted by Mr. Market seems plausible, but often it is ridiculous. The investor is free to either agree with his quoted price and trade with him, or to ignore him completely. Mr. Market doesn't mind this, and will be back the following day to quote another price. The point is that the investor should not regard the whims of Mr. Market as determining the value of the shares that the investor owns. He should profit from market folly rather than participate in it. The investor is best off concentrating on the real life performance of his companies and receiving dividends, rather than being too concerned with Mr. Market's often irrational behavior.

Charles Henry Dow (b.11/6/1851, d.12/4/1902) cofounded *Dow Jones & Company* with E. Jones and C. Bergstesser. Dow also founded *The Wall Street Journal*, which became one of the most respected financial publications in the world. He also invented the famous *Dow Jones Industrial Average* as part of his research into market movements. Furthermore he developed a series of principles for understanding and analyzing market behavior which later became known as *Dow theory*, the groundwork for *technical analysis*.

Dow published the Wall Street Journal beginning in 1889. He wrote during a period of generally rising stock prices from the depression lows in the 1870s to the then all time high in 1901. During that period Dow formulated his theory of the stock market. It consisted of two important components: the cyclical nature of the market and in the longer cycle, the "third wave", the need for confirmation between economically different sectors, specifically the industrials and the railroads. *"Nothing is more certain that the market has three well-defined movements which fit into each other. The first is the variation due to local causes and the balance of buying and selling at that particular time. The secondary movement covers a period ranging from 10 days to 60 days, averaging probably between 30 and 40 days. The third movement is the great swing covering from four to six years."*

Fundamental vs. technical analysis

Fundamental analysis maintains that markets may misprice a security in the short run but that the "correct" price will eventually be reached. Analyzing financial statements, management and competitive advantages, one can accurately estimate a “fair value” for the stock. Profits can be made by trading the mispriced security and then waiting for the market to recognize its "mistake" and reprice the security.

Technical analysis maintains that all information is reflected already in the stock price, so fundamental analysis is a waste of time. Trends 'are your friend' and sentiment changes predate and predict trend changes. Investors' emotional responses to price movements lead to recognizable price chart patterns. Technical analysis does not care what the 'value' of a stock is. Their price predictions are only extrapolations from historical price patterns.

If the markets is efficient the market price of a stock is the best possible estimate of its value. Fundamental analysis is reduced to a process which can verify that the market estimate of the value of the stock is correct. If the market is not perfectly efficient, the price of a stock can differ from its value quite considerably and fundamental analysis may be used profitably.

The efficient market hypothesis does not require that the price is always equal to the value: it is sufficient that valuation mistakes do not obey to any logic, so that they are completely random and uncorrelated so that the probability that a given stock is under/overvalued is the same at all times.

But…what should be the value of a stock?

Fischer Sheffey Black (January 11, 1938 – August 30, 1995) was an American economist, best known as one of the authors of the famous Black-Scholes equation.

"However, we might define an efficient market as one in which price is within a factor of 2 of value, i.e. the price is more than half of value and less than twice value. The factor of 2 is arbitrary, of course. Intuitively, though, it seems reasonable to me, in the light of sources of uncertainty about value and the strength of the forces tending to cause price to return to value. By this definition, I think almost all markets are efficient almost all of the time. 'Almost all' means at least 90% "

F. Black, Noise, Journal of Finance (1986) p. 533.

THE JOURNAL OF FINANCE • VOL. XLI, NO. 3 • JULY 1986

Noise

FISCHER BLACK*

ABSTRACT

The effects of noise on the world, and on our views of the world, are profound. Noise in the sense of a large number of small events is often a causal factor much more powerful than a small number of large events can be. Noise makes trading in financial markets possible, and thus allows us to observe prices for financial assets. Noise causes markets to be somewhat inefficient, but often prevents us from taking advantage of inefficiencies. Noise in the form of uncertainty about future tastes and technology by sector causes business cycles, and makes them highly resistant to improvement through government intervention. Noise in the form of expectations that need not follow rational rules causes inflation to be what it is, at least in the absence of a gold standard or fixed exchange rates. Noise in the form of uncertainty about what relative prices would be with other exchange rates makes us think incorrectly that changes in exchange rates or inflation rates cause changes in trade or investment flows or economic activity. Most generally, noise makes it very difficult to test either practical or academic theories about the way that financial or economic markets work. We are forced to act largely in the dark.

What is the "fair value" of a stock? The Gordon model

4.1 THE RATIONAL VALUATION FORMULA (RVF)

Expected Returns are Constant

One of the simplest assumptions one can make is that expected returns are constant. The expected return is defined as

$$E_t R_{t+1} = \frac{E_t V_{t+1} - V_t + D^e_{t+1}}{V_t} \tag{4.1}$$

where V_t is the value of the stock at the end of time t and D_{t+1} are dividends paid between t and $t+1$. E_t is the expectations operator based on information at time t or earlier, Ω_t. The superscript 'e' is equivalent to E_t; it helps to simplify the notation and is used when no ambiguity is likely to arise, that is:

$$D^e_{t+1} \equiv E(D_{t+1}|\Omega_t) \equiv E_t D_{t+1}$$

Assume investors are willing to hold the stock as long as it is expected to earn a constant return ($= k$). We can think of this 'required return' k as that rate of return that is just sufficient to compensate investors for the inherent riskiness of the stock:

K. Cuthbertson "Quantitative Financial Economics"

$$E_t R_{t+1} = k \quad k > 0 \tag{4.2}$$

The form of (4.2) is known as the *fair game* property of excess returns (see Chapter 5). The stochastic behaviour of $R_{t+1} - k$ is such that no abnormal returns are made, *on average*: the expected (conditional) excess return on the stock is zero:

$$E_t(R_{t+1} - k|\Omega_t) = 0 \tag{4.3}$$

Using (4.1) and (4.2) we obtain a differential (Euler) equation which determines the movement in 'value' over time:

$$V_t = \delta E_t(V_{t+1} + D_{t+1}) \tag{4.4}$$

where δ = discount factor = $1/(1+k)$ with $0 < \delta < 1$. Leading (4.4) one period:

$$V_{t+1} = \delta E_{t+1}(V_{t+2} + D_{t+1}) \tag{4.5}$$

Now take expectations of (4.5) assuming information is only available up to time t:

$$E_t V_{t+1} = \delta E_t(V_{t+2} + D_{t+2}) \tag{4.6}$$

In deriving (4.6) we have used the *law of iterated expectations*:

$$E_t(E_{t+1}V_{t+2}) = E_t V_{t+2} \tag{4.7}$$

in the future. Strictly (4.7) is an assumption that agents use *rational expectations* (see Chapter 5). Equation (4.6) holds for all periods so that

$$E_t V_{t+2} = \delta E_t (V_{t+3} + D_{t+3}), \text{ etc.} \tag{4.8}$$

By successive substitution

$$V_t = \delta D^e_{t+1} + \delta^2 D^e_{t+2} + \delta^3 D^e_{t+3} + \cdots + \delta^N (D^e_{t+N} + V^e_{t+N}) \tag{4.9}$$

Now let $N \to \infty$ and hence $\delta^N \to 0$. If the expected growth in D is not explosive so that D^e_{t+N} is finite and if V^e_{t+N} is also finite then:

$$\lim_{n \to \infty} E_t[\delta^N D_{t+N} + V_{t+N}] \to 0 \tag{4.10}$$

Equation (4.10) is known as a terminal condition or *transversality condition* and it rules out rational speculative bubbles (see Chapter 7). Equation (4.9) then becomes:

$$V_t = \sum_{i=1}^{\infty} \delta^i D^e_{t+i} \tag{4.11}$$

Where $\delta = 1/(1+k)$ we have derived (4.11) under the assumptions:

K. Cuthbertson "Quantitative Financial Economics"

- expected returns are constant
- the law of iterated expectations (i.e. *RE*) holds for all investors
- dividend growth is not explosive and the terminal condition holds
- all investors have the same view (model) of the determinants of returns and have homogeneous expectations

Expected Dividend Growth is Constant

A time series model in which (real) dividends grow at a constant rate g is the AR(1) model:

$$D_{t+1} = (1+g)D_t + w_{t+1} \tag{4.16}$$

where w_t is white noise and $E(w_{t+1}|\Omega_t) = 0$. *Expected* dividend growth from (4.16) is easily seen to be equal to g.

$$E_t D_{t+1} = (1+g)D_t$$

$$(E_t D_{t+1} - D_t)/D_t = g \tag{4.17}$$

Note that if the *logarithm* of dividends follows a random walk with drift parameter $= g$, then this also gives a constant expected growth rate for dividends (i.e. $\ln D_{t+1} = g + \ln D_t + w_t$). The optimal forecasts of future dividends may be found by leading (4.16) one period

$$D_{t+2} = (1+g)D_{t+1} + w_{t+2}$$

and using

$$E_t D_{t+2} = (1+g)E_t D_{t+1} = (1+g)^2 D_t$$

Hence by repeated substitution:

$$E_t D_{t+j} = (1+g)^j D_t \tag{4.18}$$

Substituting the forecast of future dividends from (4.18) in the rational valuation formula gives:

$$V_t = \sum_{i=1}^{\infty} \delta^i (1+g)^i D_t \tag{4.19}$$

which after some simple algebra yields

$$V_t = \frac{(1+g)}{(k-g)} D_t \quad \text{with } (k-g) > 0 \tag{4.20}$$

K. Cuthbertson “Quantitative Financial Economics”

Thus the fair value of the stock depends:

- on the current dividend D_t
- on the required return rate k
- on the expected growth rate of the dividends

Formula (4.20) is also known as Gordon formula.

Gordon, Myron J. (1959). "Dividends, Earnings and Stock Prices". *Review of Economics and Statistics* **41**: 99–105.

Weak efficiency

In weak-form efficiency excess returns cannot be made by using investment strategies based on historical prices or other historical financial data: for example it will not be possible to make excess returns by using methods such as technical analysis. A trading strategy incorporating historical data, such as price and volume information, will not systematically outperform a buy-and-hold strategy. It is often said that current prices accurately incorporate all historical information, and that current prices are the best estimate of the value of the investment. Prices will respond to news, but if this news is random then price changes will also be random. Technical analysis will not be profitable.

Assumptions behind technical analysis

- Price is determined only by supply and demand
- Supply and demand are governed by rational and irrational factors. The market continually and automatically weighs all these factors.
- Stock prices tend to move in trends which persist for an appreciable length of time.
- Changes in trend are caused by shifts in demand and supply and these shifts can be detected in the action of the market itself.

A Quantitative Approach to Tactical Asset Allocation

Journal of Wealth Management (2007) and 2009 update available at the SSRN preprint database, id1347034

MEBANE T. FABER

This article examines a very simple quantitative market-timing model. This trend following model is examined in-sample on the U.S. stock market since 1900 before out-of-sample testing across more than twenty other markets. The attempt is not to build an optimization model (indeed, the chosen model is decidedly sub-optimal, as evidenced later in the article), but to build a simple trading model that works in the vast majority of markets. The results suggest that a market timing solution is a risk-reduction technique rather than a return-enhancing one. The approach is then examined in an allocation framework since 1972, including such diverse asset classes as the Standard and Poor's 500 Index (S&P 500), Morgan Stanley Capital International Developed Markets Index (MSCI EAFE), Goldman Sachs Commodity Index (GSCI), National Association of Real Estate Investment Trusts Index (NAREIT), and United States Government 10-Year Treasury Bonds. The empirical results are equity-like returns with bond-like volatility and drawdown, and over thirty consecutive years of positive returns.

BUY RULE

Buy when monthly price > 10-month SMA.

SELL RULE

Sell and move to cash when monthly price < 10-month SMA.

1. All entry and exit prices are on the day of the signal at the close.
2. All data series are total return series including dividends, updated monthly.
3. Cash returns are estimated with 90-day commercial paper.
4. Taxes, commissions, and slippage are excluded .

Exhibit 6: S&P 500 Total Returns vs. Timing Total Returns (1900-2008)

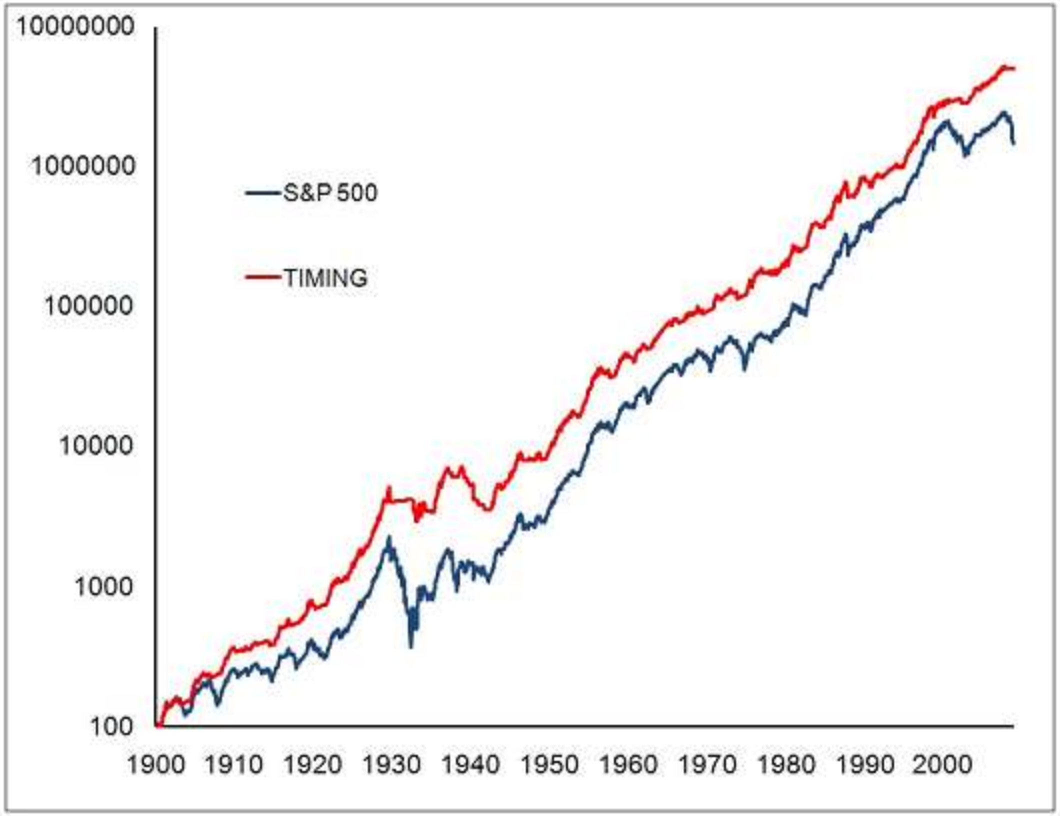


Exhibit 5: S&P 500 Total Returns vs. Timing Total Returns (1900-2008)

	S&P 500	Timing
Annualized Return	9.21%	10.45%
Volatility	17.87%	12.01%
Sharpe (4%)	0.29	0.54
Maximum drawdown	(83.66%)	(50.31%)
Best Year	52.88%	52.40%
Worst Year	(43.86%)	(26.87%)

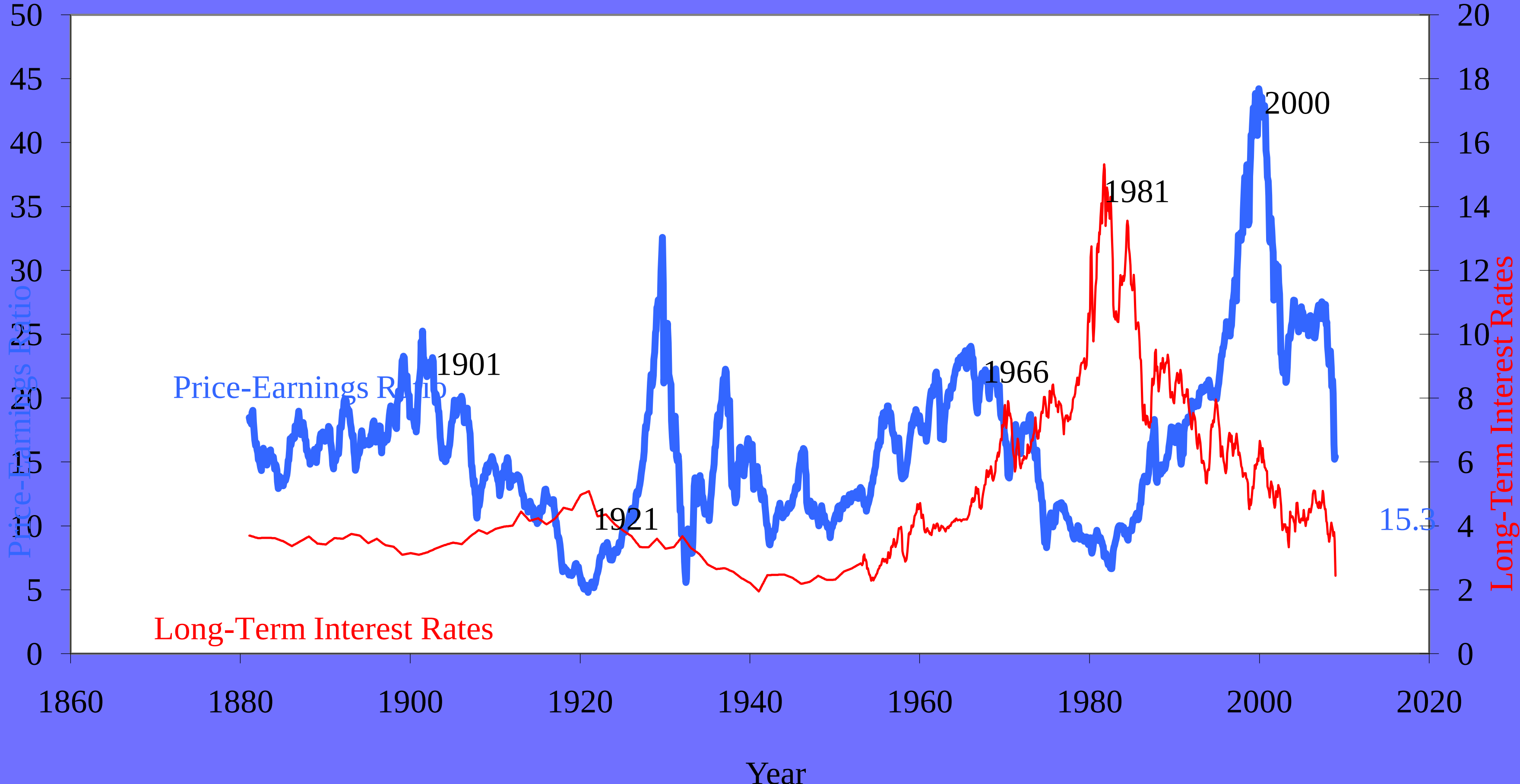
Strong and semi-strong efficiency

In the semi-strong form of the EMH a trading strategy incorporating current publicly available fundamental information (such as financial statements) and historical price information will not systematically outperform a buy-and-hold strategy. Share prices adjust instantaneously to publicly available new information, and no excess returns can be earned by using that information. Fundamental analysis will not be profitable.

In strong-form efficiency share prices reflect all information, public and private, fundamental and historical, and no one can earn excess returns. Inside information will not be profitable.

Weak, semi-strong and strong EMH

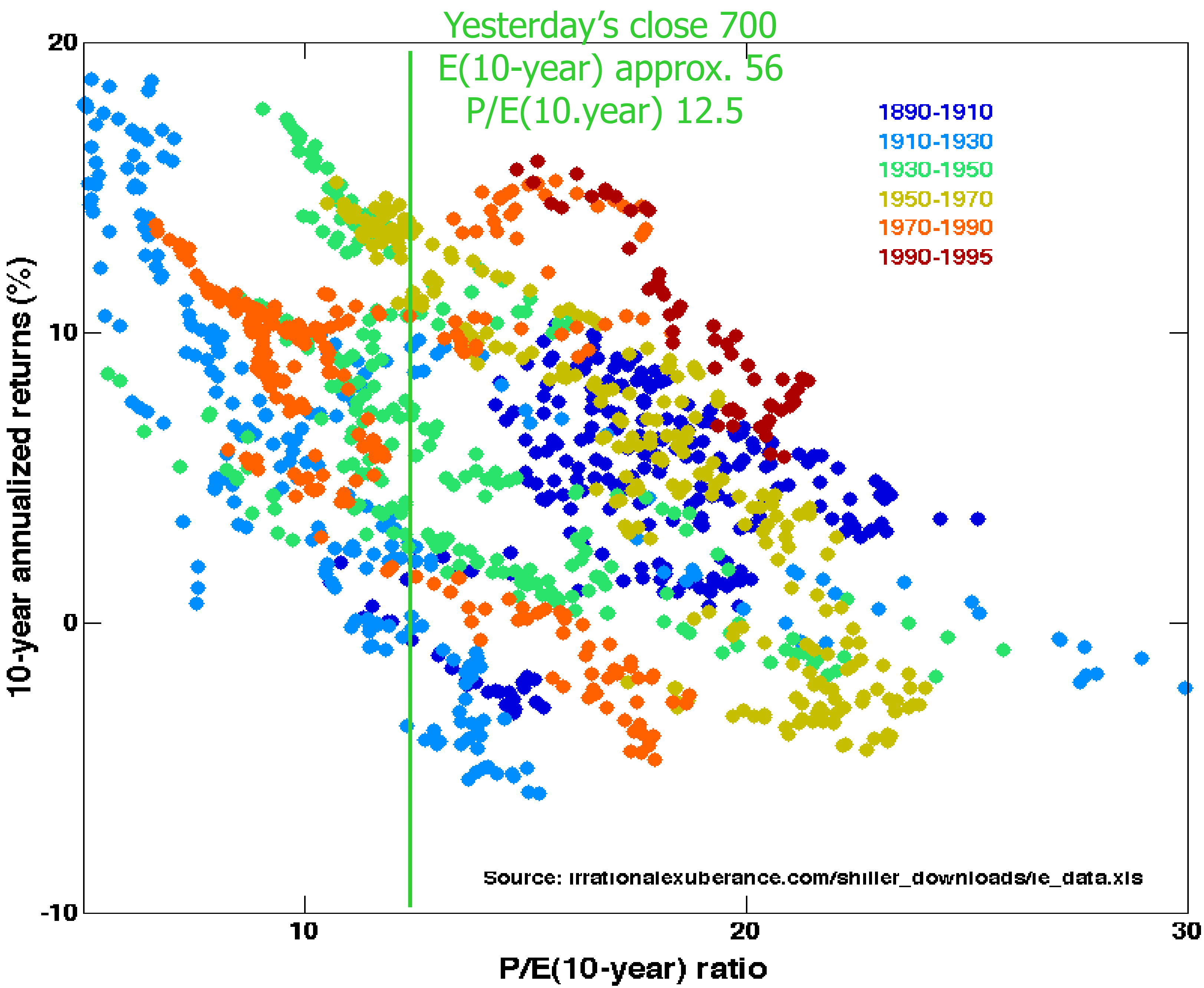
- **Weak EMH**. "One can not use *past price and volume values* to craft investment strategies that can reliably out perform the over all market."
- **Semi-Strong EMH**. "One cannot use *publically available information* to to craft investment strategies that can reliably out perform the over all market."
- **Strong EMH**. "One cannot use *any information --- including material, non-public information* --- to to craft investment strategies that can reliably out perform the over all market."



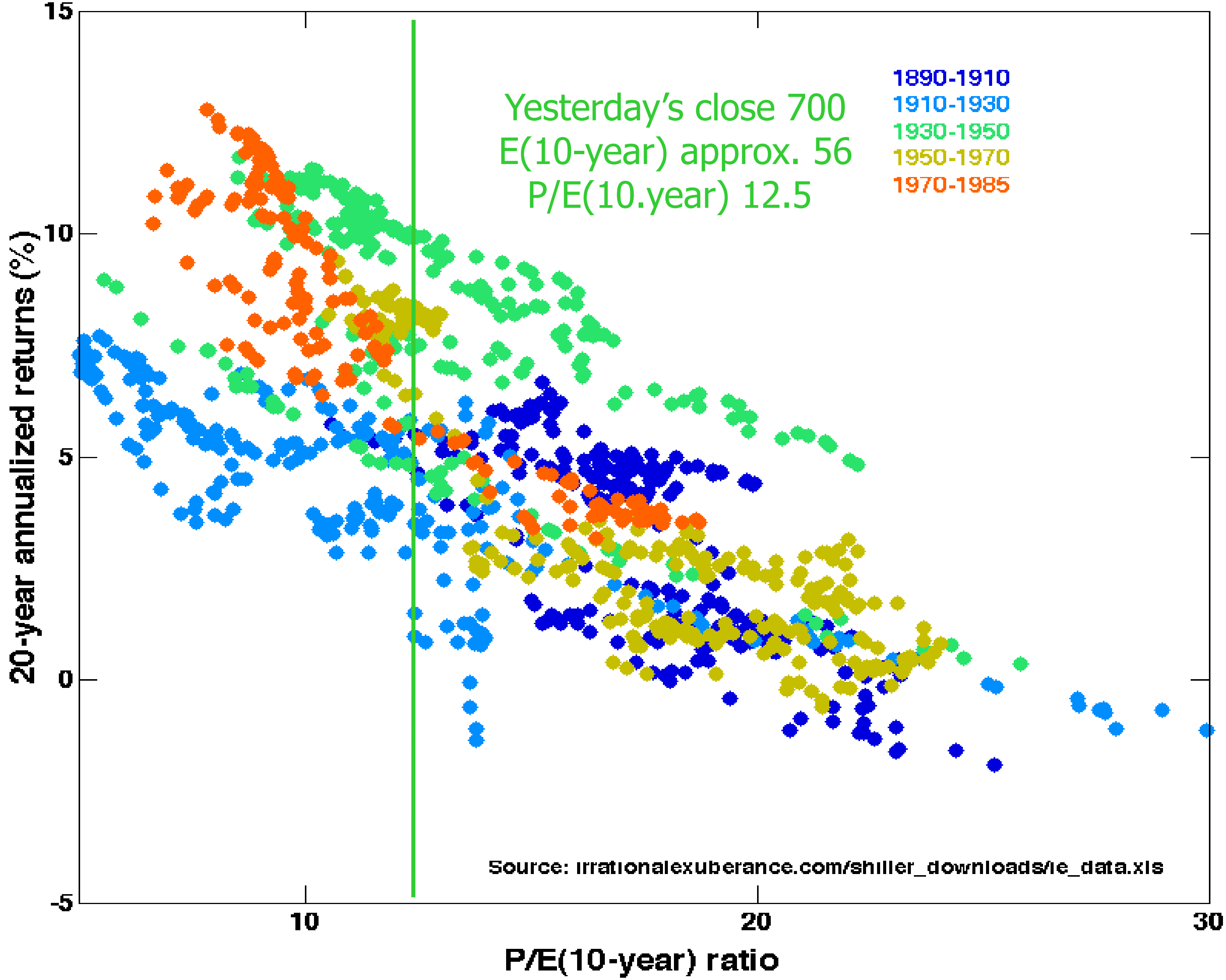
Robert Shiller's plot of the S&P Composite Real Price-Earnings Ratio and Interest Rates (1871–december 2008), from *Irrational Exuberance*, 2d ed.[1] In the preface to this edition, Shiller warns that "[t]he stock market has not come down to historical levels: the price-earnings ratio as I define it in this book is still, at this writing [2005], in the mid-20s, far higher than the historical average. … People still place too much confidence in the markets and have too strong a belief that paying attention to the gyrations in their investments will someday make them rich, and so they do not make conservative preparations for possible bad outcomes."

P/E ratios as a predictor of long term U.S. stocks returns

Price-Earnings ratios as a predictor of twenty-year returns based upon the plot by Robert Shiller (Figure 10.1 Irrational Exuberance, Princeton University Press.). The horizontal axis shows the real price/earnings ratio of the S&P500 index (inflation adjusted price divided by the prior ten-year mean of inflation-adjusted earnings). The vertical axis shows the geometric average real annual return on investing in the S&P500 index, reinvesting dividends, and selling ten or twenty years later. Data from different ten/twenty year periods is color-coded as shown in the key. According to Shiller these plots "confirms that long-term investors—investors who commit their money to an investment for ten full years—did do well when prices were low relative to earnings at the beginning of the ten years. Long-term investors would be well advised, individually, to lower their exposure to the stock market when it is high, as it has been recently, and get into the market when it is low."



Yesterday's close 700
E(10-year) approx. 56
P/E(10.year) 12.5
1890-1910
1910-1930
1930-1950
1950-1970
1970-1990
1990-1995
10-year annualized returns (%)
20
10
0
-10
10
20
30
P/E(10-year) ratio
Source: irrationalexuberance.com/shiller_downloads/ie_data.xls



Yesterday's close 700
E(10-year) approx. 56
P/E(10.year) 12.5
1890-1910
1910-1930
1930-1950
1950-1970
1970-1985
Source: irrationalexuberance.com/shiller_downloads/ie_data.xls
20-year annualized returns (%)
P/E(10-year) ratio
15
10
5
0
-5
10
20
30

Alcune analisi statistiche tuttavia indicano come, almeno in certi periodi, le variazioni di prezzo settimanali e mensili non siano completamente indipendenti dal passato e alcuni semplici indicatori fondamentali o tecnici (i multipli e il momento, tanto per citarne due dei più importanti) possano avere una qualche capacità di predire l'andamento futuro dei prezzi. Così, da almeno venti anni, la letteratura accademica si interroga sulla significatività (statistica ed economica) delle numerose *anomalie* riscontrate nelle serie storiche dei rendimenti azionari..È oggetto di controversia il fatto che gli investitori possano farne uso per ottenere dei rendimenti superiori ai benchmark. Gli investitori interessati a strategie di investimento che cerchino di utilizzare queste anomalie devono ricordare che nulla ci assicura che continueranno a prodursi in futuro: come sempre, come ben sappiamo, "i rendimenti passati *non sono garanzia* di rendimenti futuri" … Le anomalie possono scomparire perché, prive di una base economico-finanziaria, sono semplicemente frutto del *data mining*, oppure essere cancellate dall'arbitraggio compiuto da investitori (hedge funds, ad esempio) che prendono ad utilizzarle come strategie d'investimento.

Adaptive markets? (Frontiers of finance: Evolution and efficient markets, PNAS 1999)

Andrew Lo and J. Doyne Farmer have suggested that the degree of market efficiency is related to environmental factors characterizing market ecology such as the number of competitors in the market, the magnitude of profit opportunities available, and the adaptability of the market participant.

Thomas Malthus and Adam Smith—two of the forefathers of modern economics—were both cited by Darwin as inspirations for the principle of natural selection, and analogies between economics and biology have been discussed for more than a century.However, a quantitative foundation for this approach has been slow to develop.Recent research in finance suggests that this is about to change. Although there are obvious differences between evolution in biological systems and evolution in financial systems, there are also many similarities. The theory of evolution may prove to be as powerful an idea in finance as it has been in biology. There is no lack of quantitative data, and there are many opportunities for biological principles to be applied to financial interactions—after all, financial institutions are uniquely human inventions that provide an adaptive advantage to our species. This is truly a new frontier whose exploration has just begun.

Fisher Black warning concerning anomalies: *Most so-called anomalies don't seem anomalous to me at all. They seem like nuggets from a gold mine, found by one of the thousands of miners all over the world.* Si veda anche l'articolo "Noise", *Journal of Finance* vol 41, no. 3, 529-543 (1993)

The most famous "anomaly", very often recommended as a long-term investment strategy, corresponds to the so-called *value investing*. There is a quite convincing statistical evidence that investors tend to overvalue future growth prerspectives in companies with a past record with high earnings growth rates and to underestimate the future perspectives of value stocks, i.e. stocks with *low multiples* (P/E price to earnings, P/BV price to book value, P/D price to dividend, etc.) i.e. multiples which are below market and7Or sector averages

Capaul, C., I. Rowley, W.F. Sharpe "International Value and Growth Stock Returns." *Financial Analysts Journal*, vol. 49, no. 1 (January/February 1993):27-36

Fama, E.F, e K.R. French "Value versus Growth: The International Evidence." *Journal of Finance*, vol. 53, no. 6(December 1998):1975-1999.

Elroy Dimson, Paul Marsh e Mike Staunton (Triumph of the Optimists (2002, Princeton University Press) : analisi dei rendimenti di azioni e obbligazioni e dell'inflazione in 16 paesi dal 1900 al 2000. L'impatto dei dividendi e del loro reinvestimento sui rendimenti complessivi delle azioni è stato drammatico nel secolo scorso: nel caso degli Stati Uniti il rendimento annuo composto nominale (al lordo dell'inflazione e trascurando le tasse e i costi di negoziazione) delle azioni è stato del 10.1%, di cui il 5.4% imputabile al capital gain, e 4.7% all'effetto dei dividendi reinvestiti. Nel caso del mercato inglese il rendimento nominale è pure del 10.1% suddiviso in 5.1% di capital gain e 5% di effetto dividendi. Più è lungo l'orizzonte temporale di un investitore, più importante diventa il rendimento dovuto ai dividendi: su 100 anni l'effetto moltiplicatore dei dividendi corrisponde a un fattore 85 nel caso americano, 109 nel caso inglese.

Limitandosi al mercato USA, in un editoriale del Financial Analysts Journal nel 2003 Robert Arnott ha analizzato i rendimenti azionari dal 1802 al 2002. Il rendimento annualizzato delle azioni è circa il 7.9% scomponibile in 5% di rendimento dai dividendi e dal loro reinvestimento, 0.8% dall'aumento dei dividendi *reali* (al netto dell'inflazione) 1.5% dall'inflazione, 0.6% dall'aumento secolare dei multipli azionari (in questo caso l'inverso del rapporto D/P): "Dividends not only dwarf inflation, growth, and changing valuation levels individually, but they also dwarf the *combined* importance of inflation, growth, and changing valuation levels."

Returns of portfolios constructed selecting the top 30% and bottom 30% D/P ratios (1975-2006) (portfolios constructed by K. French, cfr. http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html)

	UK			HK			JAP		
	Market	High D/P	Low D/P	market	High D/P	Low D/P	market	High D/P	Low D/P
average	19.6	21.2	17.4	24.6	28.8	26.0	9.7	13.4	6.8
Standard dev.	27.1	27.9	26.5	36.9	32.4	40.6	22.6	22.8	25.4
min ret	-22.0	-21.6	-26.8	-39.4	-33.3	-49.4	-38.9	-39.1	-41.7
max ret	145.3	141.5	131.3	121.0	112.0	116.2	62.4	78.2	85.3

	IT			FR			GER		
	market	High D/P	Low D/P	market	High D/P	Low D/P	market	High D/P	Low D/P
average	18.0	20.1	16.5	17.5	22.2	13.2	13.6	15.6	12.5
Standard dev.	33.2	33.9	38.9	25.6	23.9	27.7	24.3	22.4	27.8
min ret	-31.1	-29.4	-31.5	-29.3	-21.9	-30.2	-41.0	-35.0	-51.2
max ret	111.3	109.4	123.3	62.5	65.3	65.5	76.9	65.4	93.0