Trajectories of low momentum tracks the CMS tracker

What stops the loopers in the tracker?

Loopers can either reach the calorimeters or range out in the material of the tracker. The motion along z is unaffected by the magnetic field and a relativistic particle has a z component of the velocity that is c*cosθ where θ is the polar angle.

η	sin(θ)	cos(θ)
0	1.00	0.00
0.2	0.98	0.20
0.4	0.93	0.38
0.6	0.84	0.54
0.8	0.75	0.66
1	0.65	0.76
1.2	0.55	0.83
1.4	0.46	0.89
1.6	0.39	0.92
1.8	0.32	0.95
2	0.27	0.96
2.2	0.22	0.98
2.4	0.18	0.98

Let' consider first the motion in absence of interaction with the material of the tracker.

|η|>1.5

All relativistic particles produced with |η|>1.5 reach the endcap with the same delay , independently of their transverse momentum.

1.5>|η|>0.6

Particles with pt<0.7 GeV loop in the magnetic field. Their momentum is at most 1.7 GeV. Since the endcap is positioned at three meters from the interaction point, particles with speed c*cosθ have a delay (wrt to particles produced at |η|>1.5)

$$t\_delay \approx \frac{3m}{0.3(m/ns)}\left(\frac{1}{\cos\theta}-1\right) = 10ns\left(\frac{1}{\cos\theta}-1\right)$$

From the table above we see that particles produced at |η|>0.6 arrive at the endcap (in absence of interaction) within a delay of 10 ns. So we may still consider in the same bunch crossing. The track length for such particles is between 3m and 6m.

0.6>|η|

Particles with pt<0.7 GeV loop in the magnetic field. Their momentum is at most 0.83 GeV. They curl in the field and reach the endcap at the next bunch crossing or later.

The time it takes to make a full loop is

$$t\_curl(ns) = \frac{p_t(GeV)2\pi}{c(m/\sec)\sin\theta\cdot 1.2} = 17.5\frac{p_t(GeV)}{\sin\theta}$$

and the number of loops done before reaching the endcap is

$$N\_curls = 0.6\frac{\tan\theta}{p_t(GeV)}$$

the time it takes to reach the endcap is

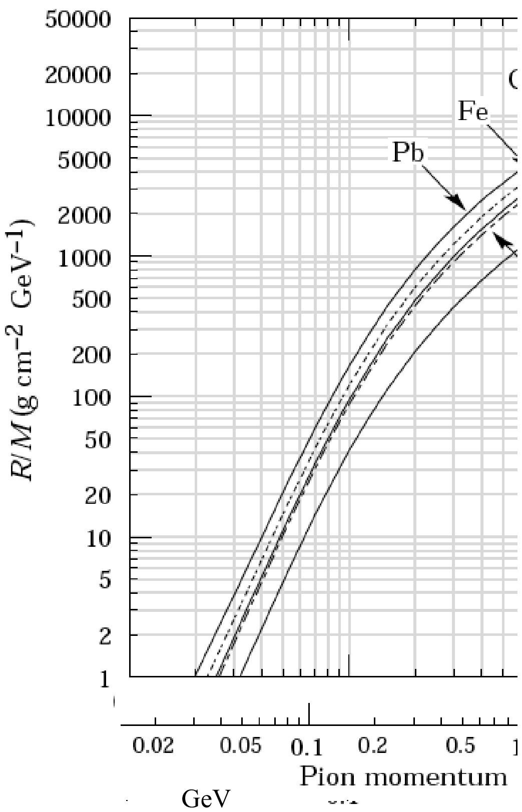
$$t\_endcap(ns) = \frac{10}{\cos\theta}$$

and the length of the trajectory is

$$l\_endcap(m) = c \cdot t\_endcap = \frac{3}{\cos\theta}$$

Material effects

The effect of the material is now considered assuming that the tracker has a uniform density. This is not a too bad approximation since the considered track lengths are longer than 1 m, while the tracker material changes on a scale smaller than 1m. The Tracker has a volume of 23.6 m^3. Assuming[1] its weight is 2.3 Tons this corresponds to an average density of 0.1 Tons/m^3 = 0.1 g/cm^3 .
The amount of material on a 1 m long trajectory is then 10 g/cm^2 that is in the right ballpark (radiation lengths of Carbon Silicon Aluminum and Copper are 43, 24 21 and 12 g/cm^2 respectively). Wolfgang Adam has verified that a particle at minimum looses in the simulation about 20 MeV/m.



A pion with momentum less than 1 GeV looses energy in collisions with electrons ($m_e << m_\pi$) and makes elastic scattering with protons and nuclei ($M >> m_\pi$) in electromagnetic and strong interactions.

An important quantity is the range of the pion that is plotted here on the left as a function of the pion momentum (http://pdg.lbl.gov/2007/reviews/passagerpp.pdf) Page 5. The relevant curve is the dot-dashed indicated by the arrow bottom right. It corresponds to a dE/dx of 2 MeV per g/cm^2.

Some relevant value of the pion ranges in the tracker material are given in the table below:

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[1] The number is roughly correct, Riccardo will compute it from the description of the Tracker in the simulation

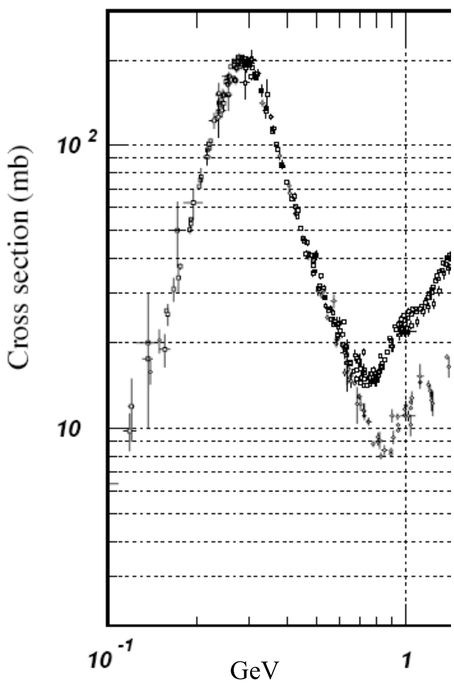
p (MeV)	Range (m)	Time (ns)	Circumference (m)	lambda nuc(m)	Eloss/turn (MeV)
700	22	73	3.7	11	73
500	14	47	2.6	5	52
300	5.6	19	1.6	1	31
200	2.8	9	1.0	4	21
100	0.4	1	0.52	-	-
50	0.04	0	0.08	-	-

In the table "Time" is simply the range divided by c: it does not include the effect of reduced velocity at the end of the range. This effect is small Δt~ 2ns. It can be computed in the following way:

$$c\Delta t + x_0 = \int_0^{x_0} \frac{dx}{\beta} = \int_{\beta\gamma_{\max}}^{0} \frac{dx}{dE}\frac{dE}{d\beta\gamma}\frac{1}{\beta} d\beta\gamma = m \int_{\beta\gamma_{\max}}^{0} \frac{dx}{dE} \cdot d\beta\gamma$$

with numerical integration of the last integral using the plot in (http://pdg.lbl.gov/2007/reviews/passagerpp.pdf) page 4 in the range $0<\gamma\beta<1$ and comparing with the range x_0 of a pion $\gamma\beta=1$. The effect is small because dx/dE is proportional to β^2 at small $\gamma\beta$.

At $0.6>|\eta|$ we can approximate p≈pt. We see that particles with p<50 MeV are simply not detected: they curl with a radius of 4 cm and they range out on a similar scale. Particles with 50 MeV<p<200 MeV range out in the tracker in less than 10 ns. They loose energy while turning and spiral down - on average - toward the center of the first circle. They will also scatter changing their directions, but since the range is smaller than the half-length of the tracker they will not reach the endcap.



The scattering plays a more important role for loopers with larger momentum: they take long time to range out and it is possible that they acquire through scattering a component of the velocity along z. This effect is relevant only if the scattering angle is larger than 30° (0.5 rad) . The scale of coulomb multiple scattering is 13MeV/p: it is not relevant for particles of few hundred MeV. Nuclear interactions however play a relevant role. The angular dependence of the cross section is flat in t (see eg BAILLON PL 50B, 387 (1974)) so that large angles are possible. The pion proton cross section in this energy region has a strong energy dependence. The interaction length in the tracker is given by:

$$\lambda(m) = \frac{M_p}{\rho\sigma} = \frac{2\cdot 10^{-24}}{0.1\cdot 100\cdot 10^{-27}} = 2\times\left(\frac{100mb}{\sigma}\right)$$

And is compiled in the table for few relevant energies. Pions with momentum around 300 MeV have an interaction length of ~ 1 m and will likely acquire a z component of the velocity. Since the half length of the tracker is 3 m they will experience multiple interactions before they reach the endcap. However loosing energy the cross section diminishes reducing the interaction probability.

More problematic is the case of loopers with momentum above 400 MeV: the nuclear interaction probability is small and the energy loss is also small. They turn loosing energy until they reach the 400~300 MeV when the scattering plays a role.
In this momentum range the energy loss is at the minimum : i.e. 2 MeV /(g/cm^2). The time needed to loose enough energy to reach 300 MeV can be computed as

$$c \cdot \Delta t = \frac{(P - 300\,MeV)}{\rho \frac{dE}{dx}} \quad \text{that gives} \quad \Delta t(ns) = \frac{P(MeV) - 300}{6}$$

i.e a pion of 600 MeV will take 50 ns to slow down to 300 MeV when it will experience a nuclear scattering with high probability.

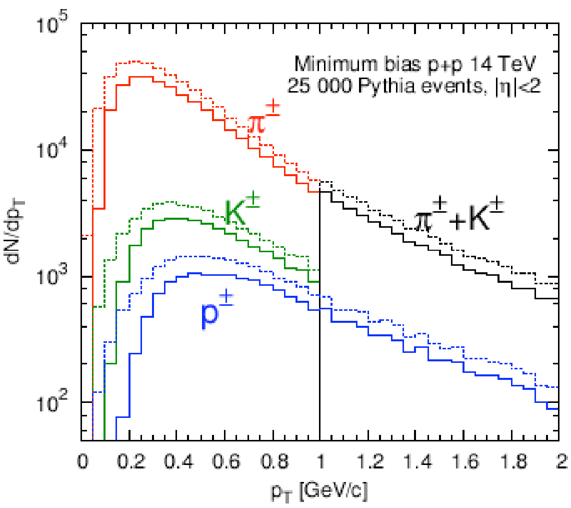
Summary

There is a region of the phase space of low momentum particles:
$-0.6<\eta<0.6$ and 300 MeV<pt<700 MeV
where there is no fast mechanism to stop loopers. They can stay trapped in the Tracker for long time: up to 75 ns before they range out. It will be interesting to compare these simple observations with the simulation.

Appendix

The plot from
https://twiki.cern.ch/twiki/pub/CMS/SoftPhysics/pp_spectra.gif

shows important features of minbias events:



the pt distribution has a maximum at pt ~ 0.25 GeV and decreases somewhat exponentially. The maximum reflects the fact that pt is the length of a vector in a two dimensional space and the phase space go to zero for the length that goes to zero
(i.e. dpx dpy=pt dpt dphi).