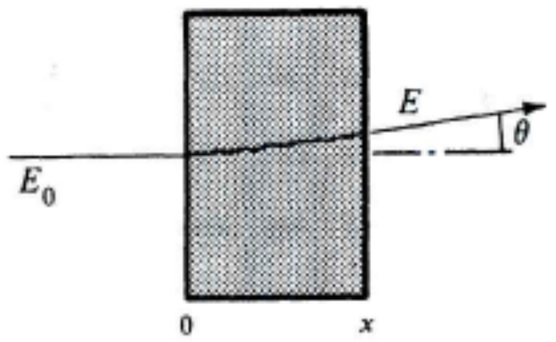
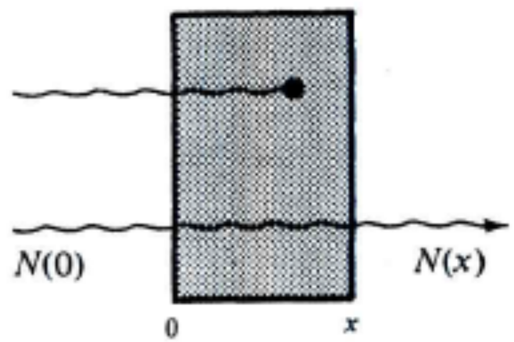
Passage of Particle through matter

Early X rays pioneers burnt their hands
All early cyclotron physicist had cataracts
YES… particles do interact with matter

Consider a well collimated monochromatic beam of particles passing through a slab : two extreme cases



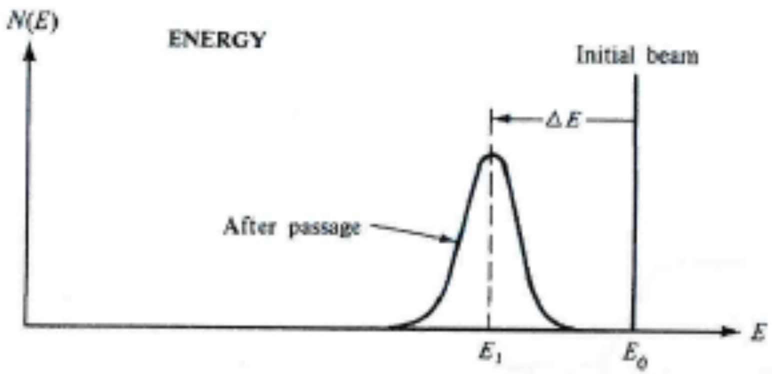
Many small interactions



One “large” interaction

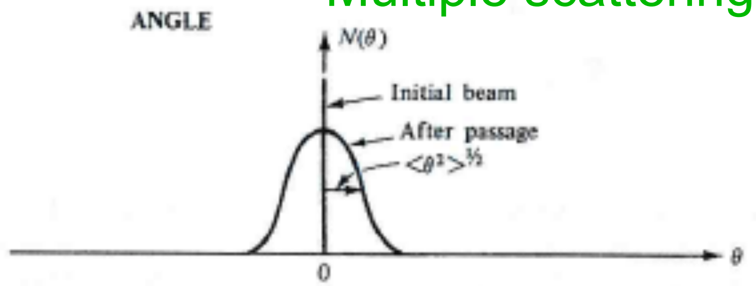
Gigi.Rolandi@cern.ch

Many small events

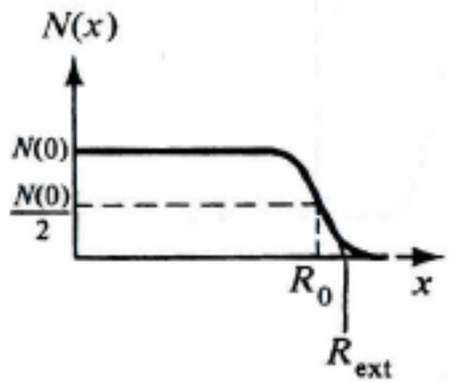


Energy loss

Multiple scattering



Range



Up to a certain thickness of the absorber all particle are transmitted , though with degraded energy and direction

Gigi.Rolandi@cern.ch

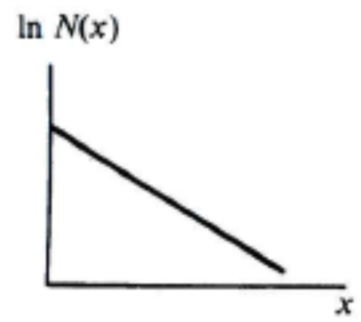
All or Nothing interaction

If the transmitted particles have not undergone an interaction, the transmitted beam has the same energy and angular spread of the initial one, simply it is less intense. In each elementary slab dx the number of interaction is proportional to the number of incoming particles

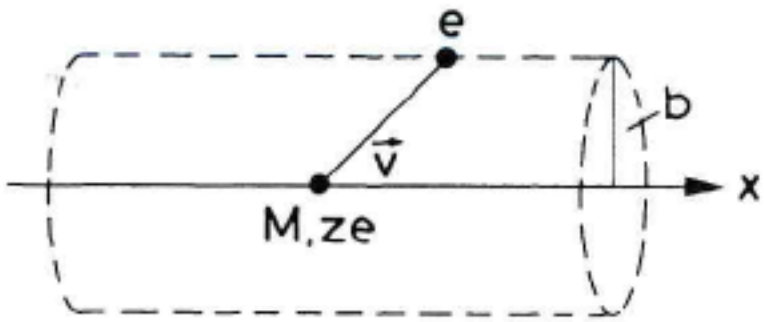
$$dN = -N(x) \cdot \mu \cdot dx$$

$$N(x) = N(0) \cdot e^{-\mu x}$$

No range can be defined. The average distance traveled by a particle before going interaction is called MEAN FREE PATH and is 1/μ



Heavy Charged particles: Energy loss



Consider a charge particle (M>>m) passing with impact parameter b from one electron considered at rest "during the interaction" .

The electron will get some momentum from the particle:

$$I = \int F\,dt = e\int E_\perp\,dt = e\int E_\perp \frac{dt}{dx}\,dx = e\int E_\perp \frac{dx}{v}$$

The integral can be calculated with Gauss

$$\int E_\perp 2\pi b\,dx = 4\pi z e\,, \qquad \int E_\perp\,dx = \frac{2ze}{b}$$

Gigi.Rolandi@cern.ch

Energy loss (2)

The energy gained by the electron is $\Delta E(b) = \frac{I^2}{2m_e} = \frac{2z^2e^4}{m_e v^2 b^2}$

If we consider now N_e electron per unit volume , the energy lost by the particle to electrons in the volume dV= 2πb db dx is

$$-dE(b) = \Delta E(b)\, N_e\, dV = \frac{4\pi z^2 e^4}{m_e v^2} N_e \frac{db}{b} dx$$

Gigi.Rolandi@cern.ch

Energy loss (3)

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{b_{max}}{b_{min}}$$

What are b_{min} and b_{max} ? They must be values where our initial assumptions are verified.

Max energy transfer to an electron is $2\gamma^2 m_e v^2$ Dimostrare

That implies $b_{min} = \frac{z e^2}{\gamma m_e v^2}$

Gigi.Rolandi@cern.ch

Energy loss (4)

b_{max} is more complex: in order to transfer energy to a bound electron the perturbation time must be small respect to the orbit period $1/\nu$ otherwise there is no energy transfer. Since the interaction time is $t=b/(\gamma v)$ then

$$b_{\max} = \frac{\gamma v}{\bar{\nu}}$$

And we obtain then

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{\gamma^2 m v^3}{z e^2 \bar{\nu}}$$

Gigi.Rolandi@cern.ch

Bethe Bloch

$$\frac{dE}{dX} = -K \frac{Z}{A} \frac{\rho}{\beta^2} \left\{ \ln \frac{2mc^2\beta^2 E_M}{I^2(1-\beta^2)} - 2\beta^2 \right\}, \qquad K = \frac{2\pi N z^2 e^4}{mc^2} = 0.154 \text{ MeV g}^{-1} \text{ cm}^2$$

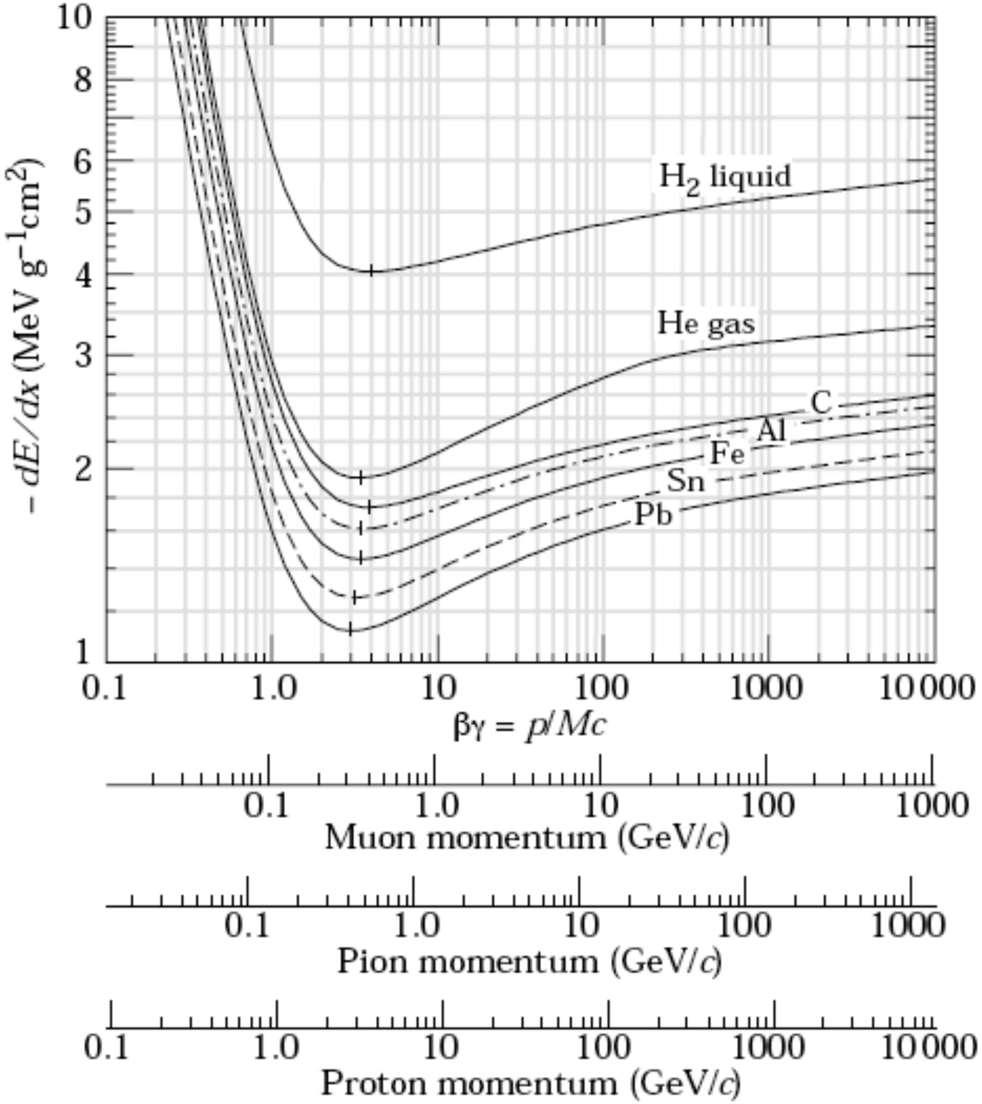
$E_M = 2\gamma^2 mv^2$ is the maximum electron energy
$I \sim I_o Z$ is the effective ionization potential , I_o=12 eV

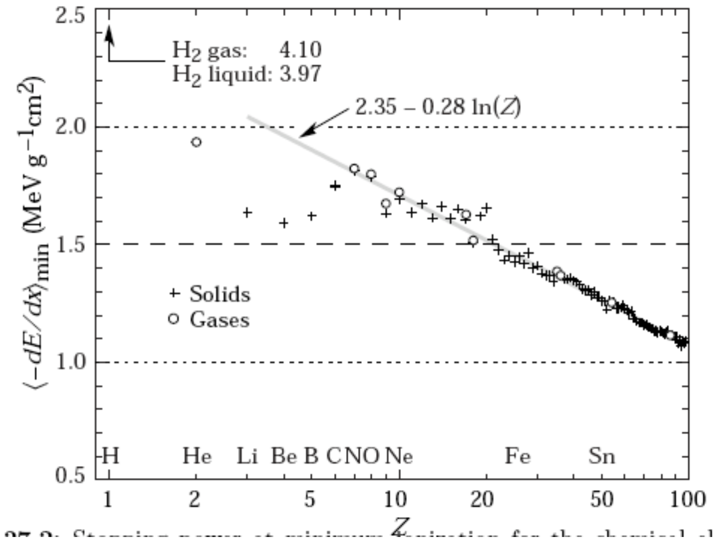
In practical application the thickness of the absorber is not measured in thickness dx but in “material thickness “ ρdx, where ρ is the material density

$$\frac{dE}{d(\rho x)} = \frac{1}{\rho}\frac{dE}{dx}.$$

Gigi.Rolandi@cern.ch

Bethe Bloch





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dE/dx as particle identification

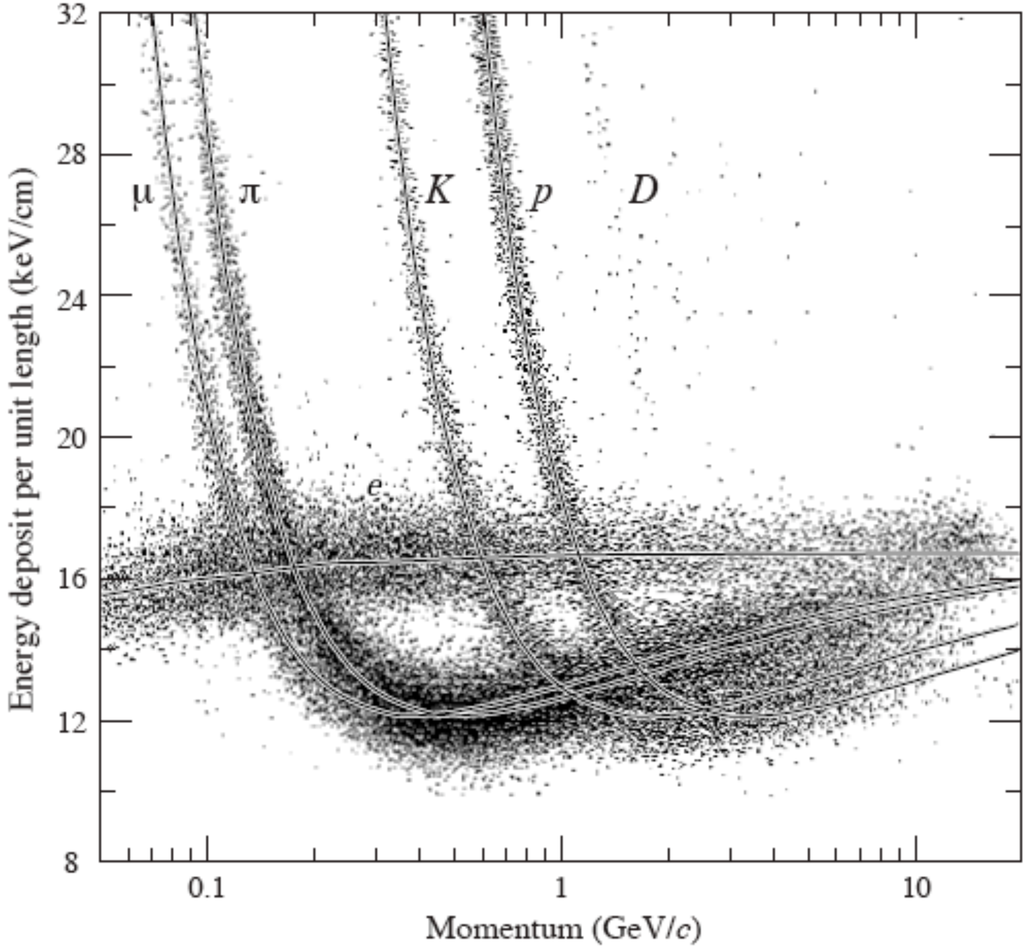
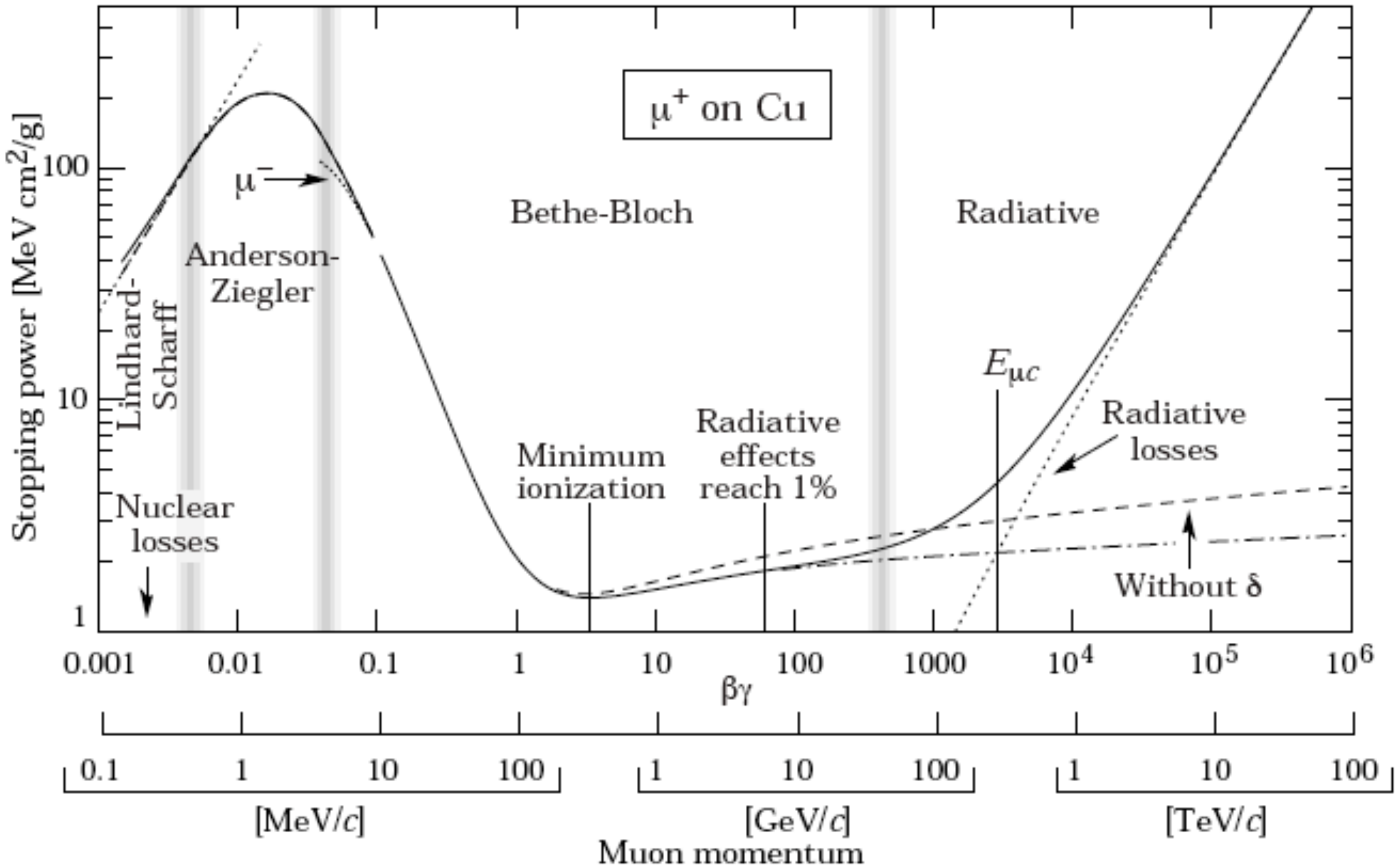
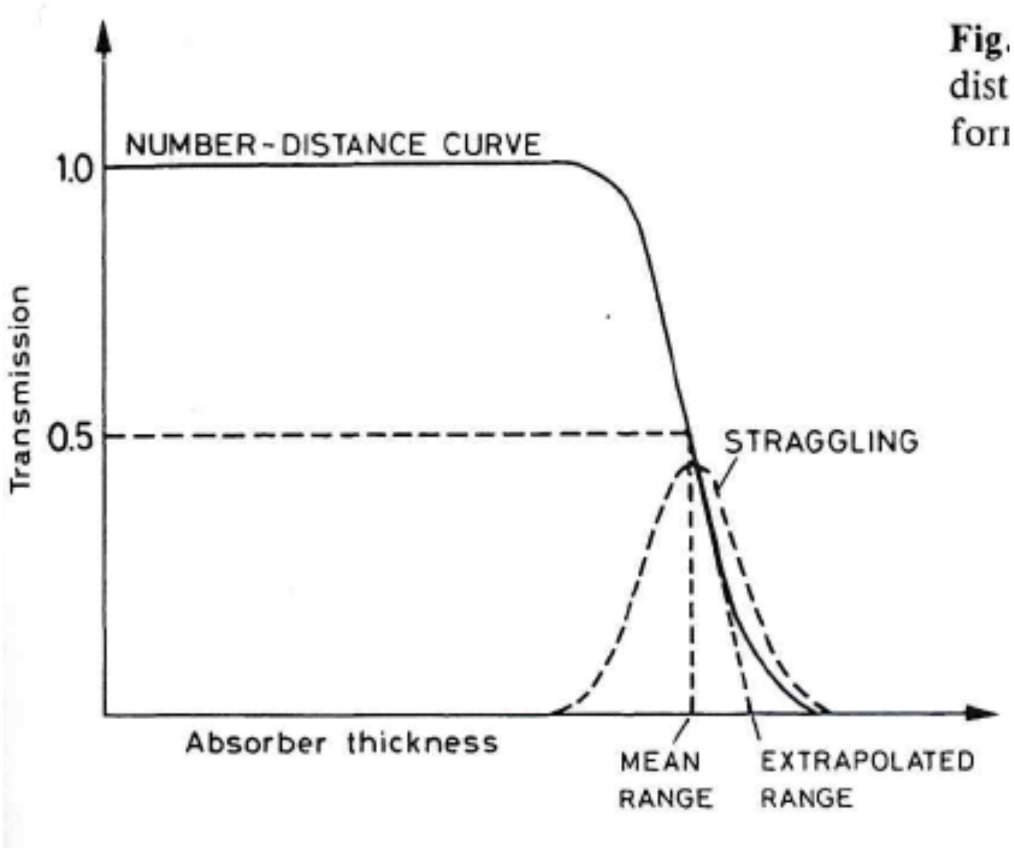


Figure 28.8: PEP4/9-TPC energy-deposit measurements (185 samples @8.5 atm Ar-CH_4 80–20%) in multihadron events. The electrons reach a Fermi plateau value of 1.4 times the most probably energy deposit at minimum ionization. Muons from pion decays are separated from pions at low momentum; π/K are separated over all momenta except in the cross-over region. (Low-momentum protons and deuterons originate from hadron-nucleus collisions in inner materials such as the beam pipe.)

Extending the Bethe Bloch



Range



$$S(T_0) = \int_0^{T_0} \left(\frac{dE}{dx}\right)^{-1} dE$$

Gigi.Rolandi@cern.ch

Range

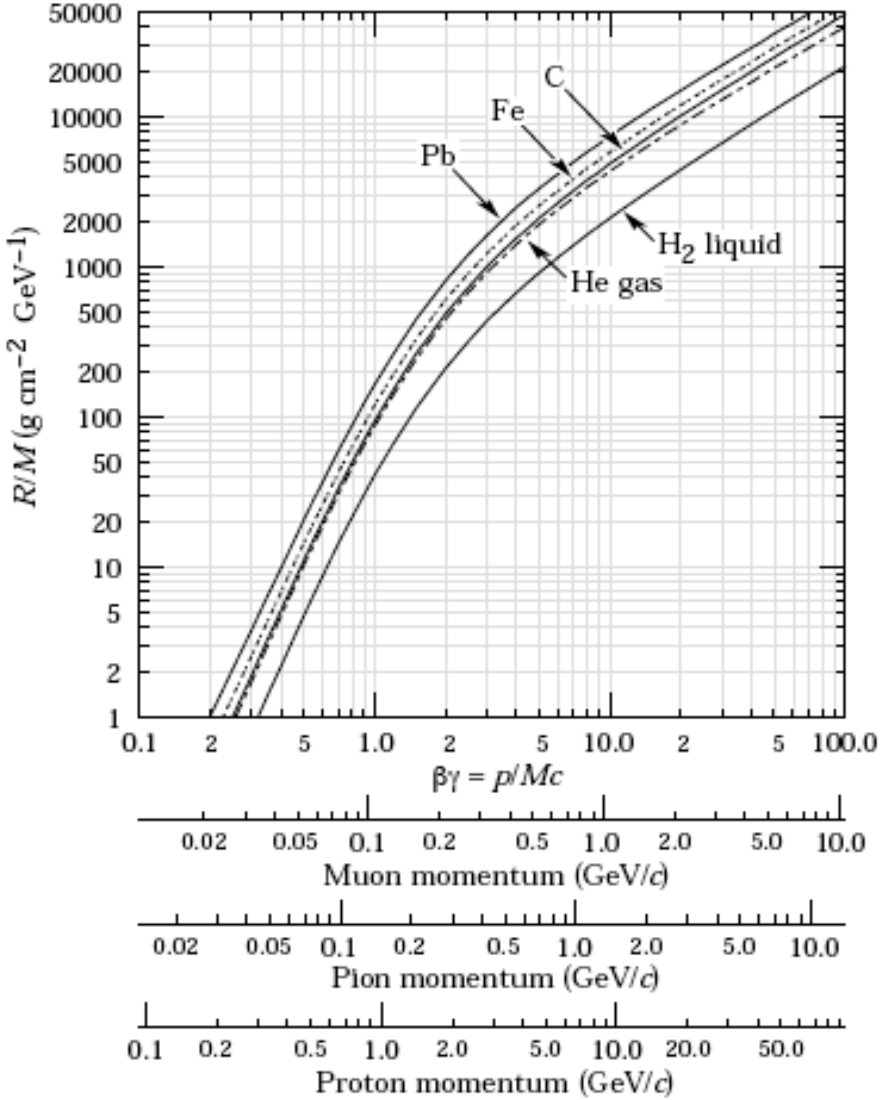


Figure 27.4: Range of heavy charged particles in liquid (bubble chamber) hydrogen, helium gas, carbon, iron, and lead. For example: For a K^+ whose momentum is 700 MeV/c, $\beta\gamma = 1.42$. For lead we read $R/M \approx 396$, and so the range is 195 g cm^{-2}.

Struggling in the energy loss

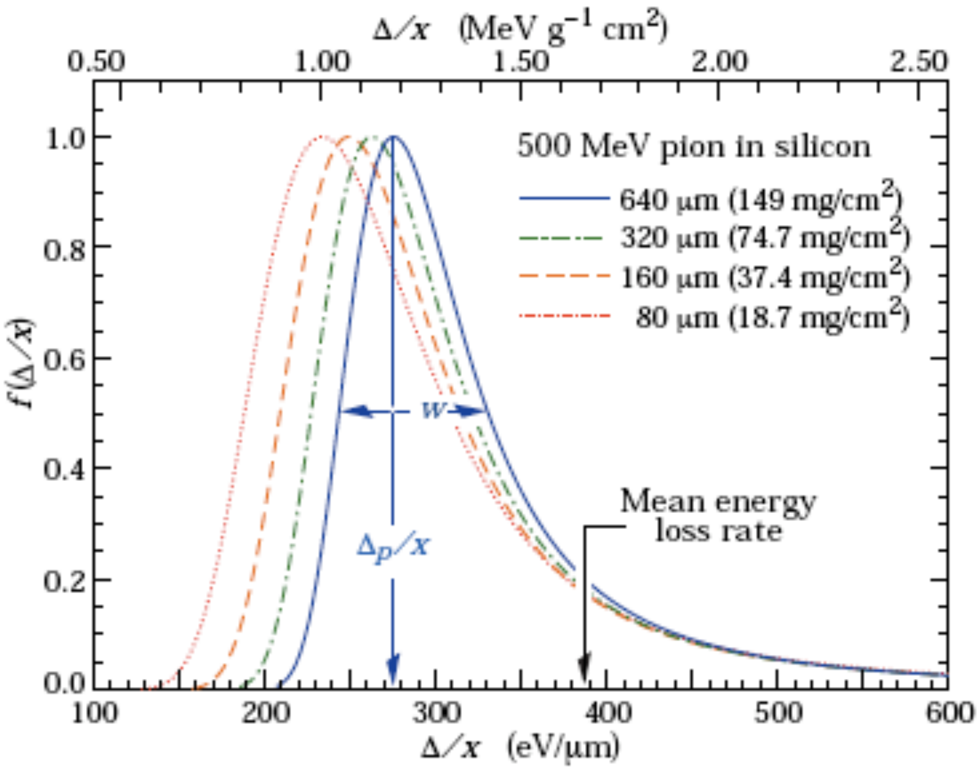


Figure 27.7: Straggling functions in silicon for 500 MeV pions, normalized to unity at the most probable value δ_p/x. The width w is the full width at half maximum. See full-color version on color pages at end of book.

Gigi.Rolandi@cern.ch

Esercizi

- A proton beam of 100 MeV has to be slow down to 50 MeV. Compute the thickness of a lead and carbon absorber in cm and in gr cm^{-2}
- What is the length of the beam dump of the SPS (450 GeV proton). Are we neglecting something in our calculation ?
- Cosmic muons are observed in underground laboratories at 2-3 km depth. What is the original momentum of the muons? Why protons are not observed?
- A beam 1 mA passes through 1 cm^3 of copper. Computer the energy deposited per second and (assuming it to be isolated) the temperature rise
- Design a beam line that produces protons of 500 MeV +/- 0.05% from a proton beam of 1 GeV .

Gigi.Rolandi@cern.ch

Different types of energy loss

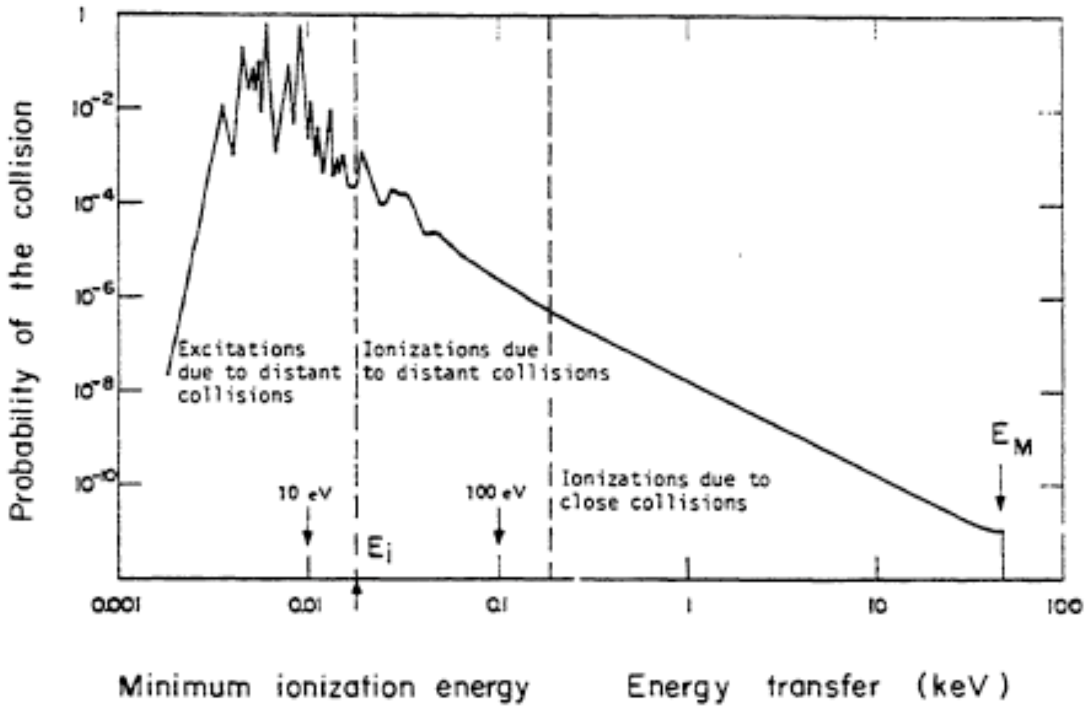


Fig. 2 Relative probability of different processes induced by fast (100 keV) electrons in water, as a function of the energy transfer in a collision[4]. The maximum kinematically allowed energy transfer, E_M = 50 keV in this case, is also shown.

Close collisions with large energy loss, distant collisions resulting in ionization or excitation (frequently followed by photon emission)

Gigi.Rolandi@cern.ch

Delta rays

Close collisions produce "high" energy electrons. The probability for such an event is essentially the first term of the Bethe-Bloch formula

$$P(E) = K \frac{Z}{A} \frac{\rho}{\beta^2} \frac{X}{E^2}$$

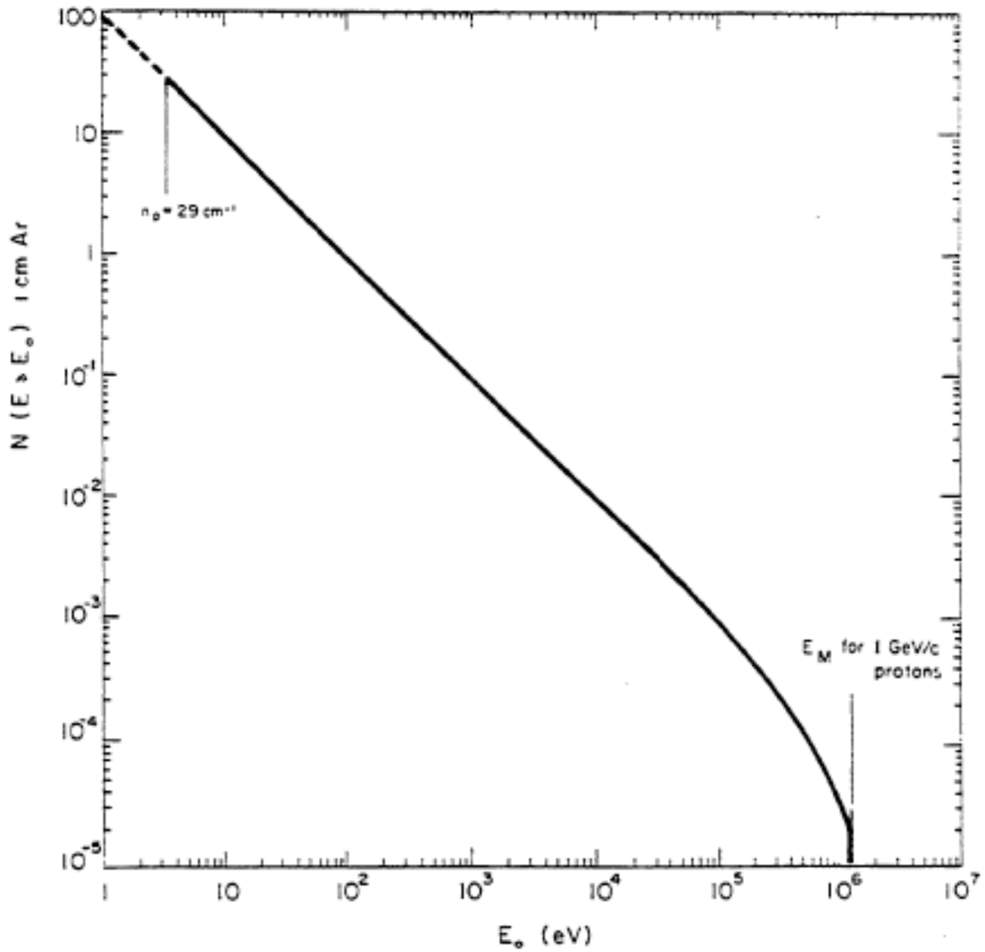
That can be rewritten as

$$P(E)dE = \frac{K}{\beta^2} \frac{Z}{A} \frac{x}{E^2} dE = W \frac{dE}{E^2}$$

The number of electrons with energy larger than E_o is

$$N(E \geq E_0) = \int_{E_0}^{E_M} P(E)\, dE = W\left(\frac{1}{E_0} - \frac{1}{E_M}\right) \simeq \frac{W}{E_0}$$

Delta Rays (2)



Computed number of δ electrons ejected at an energy larger than or equal to E_0, as a function of E_0, in 1 cm of argon at normal conditions. The average number of primary ionizing collisions (29 per cm) and the maximum allowed energy transfer for 1 GeV/c protons are shown.

Gigi.Rolandi@cern.ch

Delta rays (3)

The angle of emission of a delta ray wrt the direction of the ionizing particle is given by

$$\cos^2 \theta = \frac{E}{E_M}$$

However due the very large electron-molecule cross section (~ 10^{-16} cm^2 for E ~ keV) the direction of the delta ray is quickly randomized.

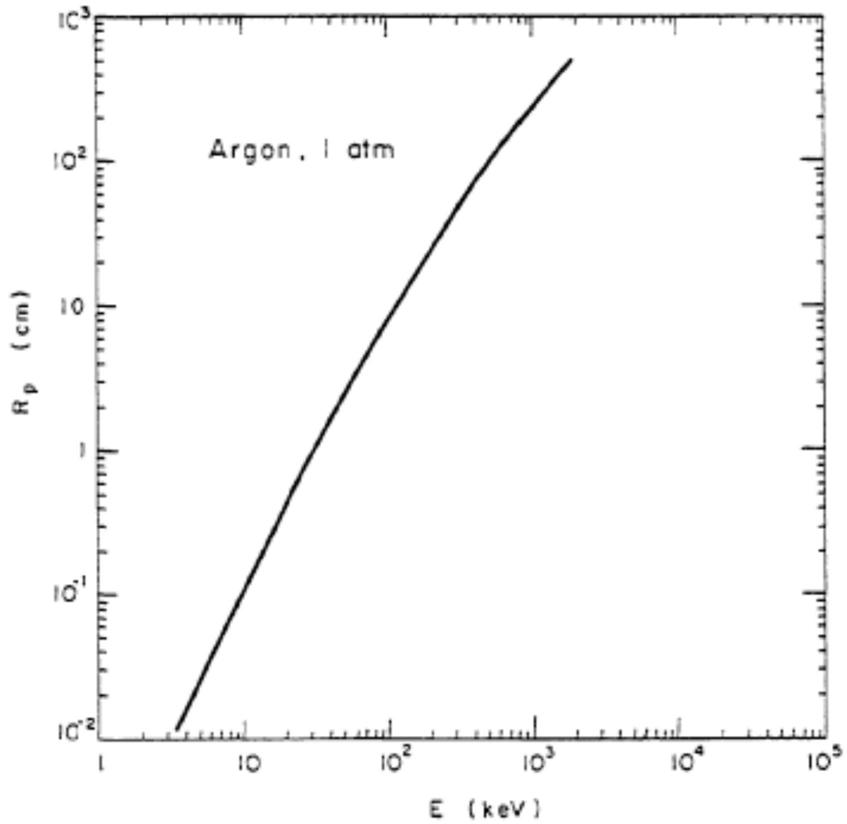
Gigi.Rolandi@cern.ch

Range of Delta Rays (4)

Delta rays have a quite random motion due to scattering with molecules. Their range can be computed integrating the Bethe-Bloch curve, however, this gives a over-estimation of the distance travelled from the track because of the zig-zag motion. A practical range (that is 2-3 times smaller than the BB) is given by the expression:

$$R_p = 0.71\, E^{1.72} \qquad (E \text{ in MeV})$$

R_p in gr cm^{-2}

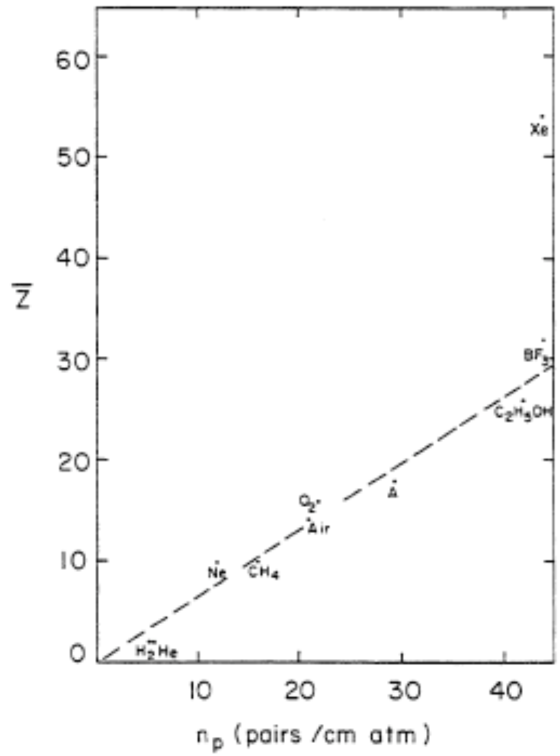


g. 5 Range of electrons in argon, at normal conditions as a function of energy, deduced from measurement in light materials[8])

Primary and secondary ionization

When an heavy particles passes through a medium it will produce a number of PRIMARY ionization events interacting with Nuclei. Some of these primary electrons have “high” energy and will produce further ionization events. The sum of the primary and secondary ionization is called TOTAL ionization.

$$n_T = \frac{\Delta E}{W_i}$$



Gigi.Rolandi@cern.ch

Table 1

Properties of several gases used in proportional counters (from different sources, see the bibliography for this section). Energy loss and ion pairs per unit length are given at atmospheric pressure for minimum ionizing particles

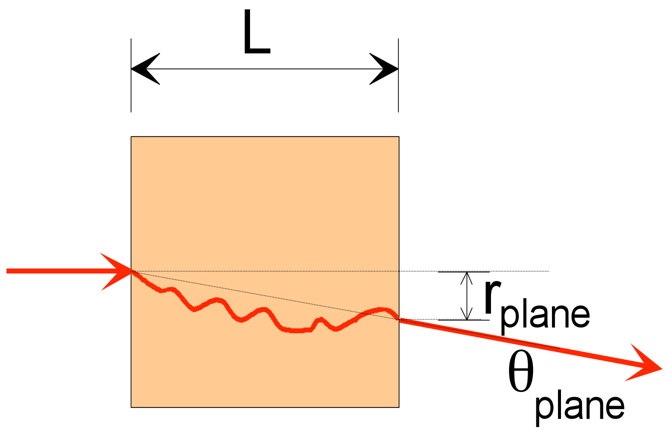
Gas	Z	A	δ (g/cm³)	E_{ex} (eV)	E_i (eV)	I_0 (eV)	W_i	dE/dx (MeV/g cm⁻²)	dE/dx (keV/cm)	n_p (i.p./cm) a)	n_T (i.p./cm) a)
H_2	2	2	8.38×10^{-5}	10.8	15.9	15.4	37	4.03	0.34	5.2	9.2
He	2	4	1.66×10^{-4}	19.8	24.5	24.6	41	1.94	0.32	5.9	7.8
N_2	14	28	1.17×10^{-3}	8.1	16.7	15.5	35	1.68	1.96	(10)	56
O_2	16	32	1.33×10^{-3}	7.9	12.8	12.2	31	1.69	2.26	22	73
Ne	10	20.2	8.39×10^{-4}	16.6	21.5	21.6	36	1.68	1.41	12	39
Ar	18	39.9	1.66×10^{-3}	11.6	15.7	15.8	26	1.47	2.44	29.4	94
Kr	36	83.8	3.49×10^{-3}	10.0	13.9	14.0	24	1.32	4.60	(22)	192
Xe	54	131.3	5.49×10^{-3}	8.4	12.1	12.1	22	1.23	6.76	44	307
CO_2	22	44	1.86×10^{-3}	5.2	13.7	13.7	33	1.62	3.01	(34)	91
CH_4	10	16	6.70×10^{-4}		15.2	13.1	28	2.21	1.48	16	53
C_4H_{10}	34	58	2.42×10^{-3}		10.6	10.8	23	1.86	4.50	(46)	195

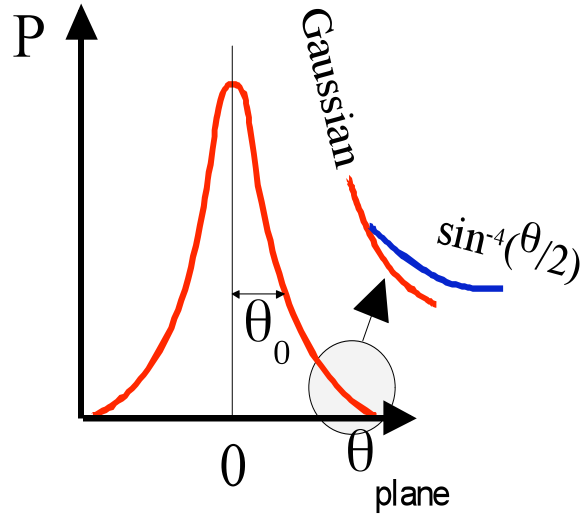
a) i.p. = ion pairs

Gigi.Rolandi@cern.ch

Multiple Scattering (1)

Sufficiently thick material layer → the particle will undergo multiple scattering.





$$\theta_0 = \theta_{plane}^{RMS} = \sqrt{\langle \theta_{plane}{}^2 \rangle} = \frac{1}{\sqrt{2}} \theta_{space}^{RMS}$$

Approximation :

$$\theta_0 \propto \frac{1}{p} \sqrt{\frac{L}{X_0}}$$

X_0 is radiation length of the medium

Multiple scattering (2)

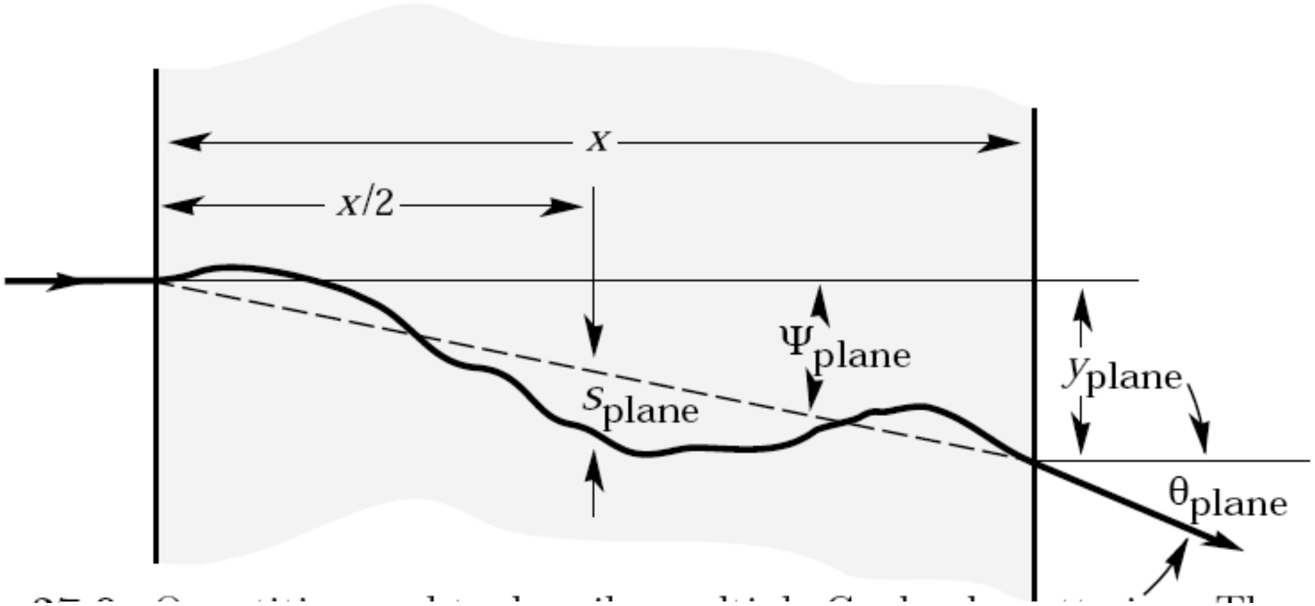
$$\theta_0 = \frac{13.6 \text{ MeV}}{\beta c p} \, z \, \sqrt{x/X_0}\Big[1 + 0.038 \ln(x/X_0)\Big]$$

$$\frac{1}{2\pi\,\theta_0^2} \exp\left(-\frac{\theta_{\text{space}}^2}{2\theta_0^2}\right) d\Omega \;,$$

$$\frac{1}{\sqrt{2\pi}\,\theta_0} \exp\left(-\frac{\theta_{\text{plane}}^2}{2\theta_0^2}\right) d\theta_{\text{plane}} \;,$$

Gigi.Rolandi@cern.ch

Multiple Scattering (3)



$$\psi_{\text{plane}}^{\text{rms}} = \frac{1}{\sqrt{3}}\,\theta_{\text{plane}}^{\text{rms}} = \frac{1}{\sqrt{3}}\,\theta_0 \;,$$

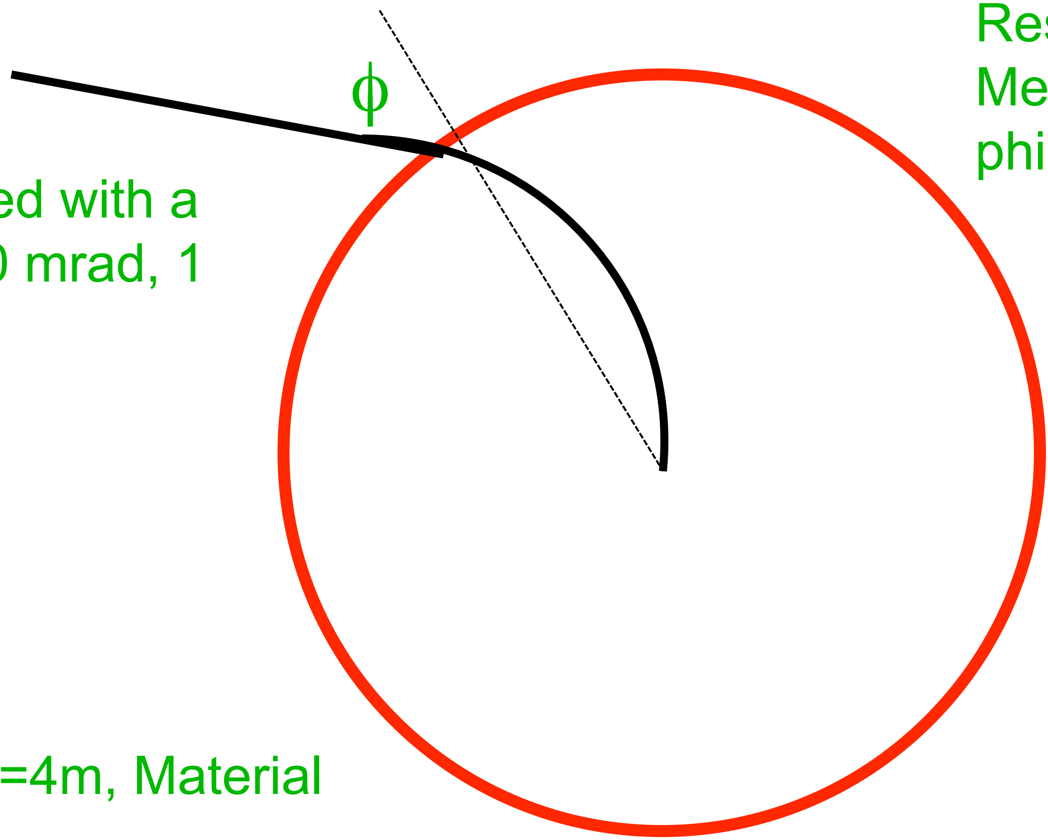
$$y_{\text{plane}}^{\text{rms}} = \frac{1}{\sqrt{3}}\,x\,\theta_{\text{plane}}^{\text{rms}} = \frac{1}{\sqrt{3}}\,x\,\theta_0 \;,$$

$$s_{\text{plane}}^{\text{rms}} = \frac{1}{4\sqrt{3}}\,x\,\theta_{\text{plane}}^{\text{rms}} = \frac{1}{4\sqrt{3}}\,x\,\theta_0 \;.$$

Gigi.Rolandi@cern.ch

Momentum resolution in "CMS"

Compute momentum Resolution from the Measurement of the phi angle.



Phi is measured with a precision of 10 mrad, 1 mrad. Discuss

B field 4T, R=4m, Material 120 X0.

Gigi.Rolandi@cern.ch