The Growth of Firms

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di Studi Universitari e di Perfezionamento

Common Dissatisfaction

Lack of relation between theoretical and empirical investigations. . .

Economics consists of theoretical laws which nobody has verified

and of empirical laws which nobody can explain.

The Main Bulding block: Economic Distribution Laws

Economic distributions as steady-state equilibrium:

- certain economic distributions are stable over time
- we are aware of a continuing movement of the elements which make up the population in question

This suggests the idea of steady-state equilibrium: “a state of macroscopic equilibrium maintained by a large number of transitions in opposite directions” (Feller, 1957)

An Example

Let us consider two “economic” populations

HUMAN BEINGS	FIRMS
total population	total number of firms
age-structure	size distribution
expected birth and death rates	expected gains/losses or ruin plus entry

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Outline

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Log-normal behaviour

Let S_i the size of firm i and let $s_i = \log(S_i)$ its log,

$$f_s(x) = \text{Prob}\left\{x < s < x + dx\right\}/dx = \frac{1}{\sqrt{2\,\pi\,\sigma^2}}\exp(-\frac{(x-\mu)^2}{2\,\sigma^2}).$$

Then the log-density has a parabolic behaviour

$$\log(f_s(x)) \sim -(x-\bar{x})^2$$

Growth rates are uncorrelated with size, their distribution is the same for small and large firms

$$f_{g,s}(x,y) = f_g(x)\,f_s(y)\ .$$

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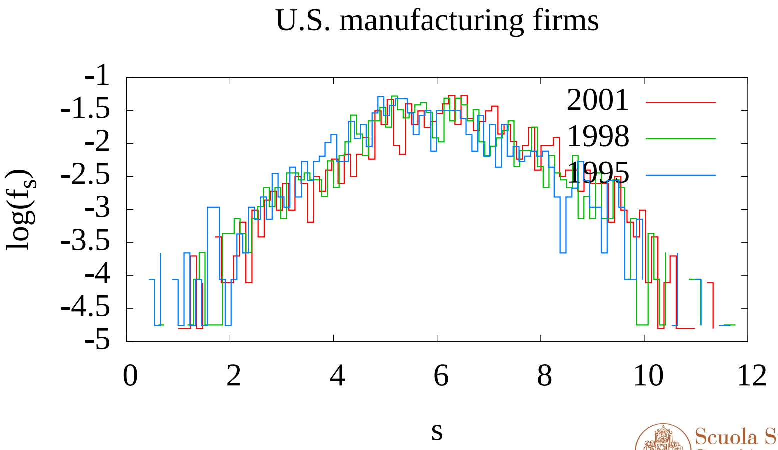
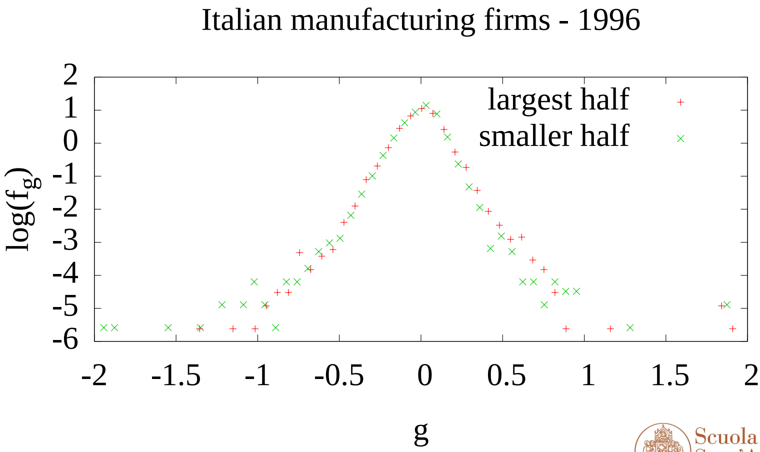
COMPUSTAT aggregate size distribution



ISTAT aggregate binned growth rates density



Kapteyn and Gibrat

Firm size distribution conforms approximately to normality once plotted on a log scale, how this distribution arises?

Kapteyn and Gibrat started from independence of growth rates on size. They proposed the Law of Proportionate Effect: equal proportionate increments have the same chance of occurring in a given time-interval whatever size happens the firm to have reached.

At a given time t growth is proportional to size

$$S_t - S_{t-1} = \eta_t \, S_{t-1} \quad \eta_t \quad \text{independent} \quad S_{t-1}$$

In log the relation is additive

$$s_t - s_{t-1} = \epsilon_t \quad \epsilon \sim i.i.d.(\mu, \sigma^2)$$

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The Law of Proportionate Effect

Law's strong version: increments are independent and identically distributed, $\epsilon_t \sim i.i.d.(\mu, \sigma^2)$.

Law's weak version: ϵ_t follow a stationary (possibly correlated) process.

Intertemporal iteration leads to an integrated process

$$s(t+T) = s(t) + \epsilon(t) + \epsilon(t) + \ldots + \epsilon(T-1) \ .$$

Laws's strong version is analogous to a geometric Brownian motion, i.e. a *diffusion* in logs.

Central limit theorem: both strong and (almost) weak Law's give

$$\lim_{t \to +\infty} S_T \ \text{normal} \ (T\,\mu, T\,\sigma^2) \ .$$

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Weakness of the Gibrat's Model

At this point two main weaknesses of the "unrestricted" Gibrat's model deserve to be highlighted

- theoretically, the variance of size $t\sigma^2$ explodes for $t \to \infty$
- empirically, we do not observe any increase in the dispersion of the size distribution

To cope with this drawback it is necessary to introduce a stability condition to offset the tendency to diffusion. the type of condition chosen and its interpretation become the distinctive feature of various theories emanating from the Gibrat's model.

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Enter Kalecki

Kalecki started from the observation that the variance of the size of all business firms remains constant over time

$$\frac{1}{N}\sum(s_t + g_t)^2 = \frac{1}{N}\sum(s_t)^2 \quad ,$$

$$\Downarrow$$

$$2\sum s_t\, g_t = -\sum g_t^2 \ . \qquad (1)$$

To assure stability of size-distribution, the random increment should be negatively correlated with size.

He assumed a linear relation between g_t and X_t

$$g_t = -\alpha s_t + z_t \ , \qquad (2)$$

and proved that the distribution of S_t is a Normal.

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Behaviour of the upper tail

Let s_i the size of firm i and let $s_i = \log(S_i)$ its log,

$$F_s(x) = \text{Prob}\{s \leq x\} = \text{fraction of firms with log(size)} \leq x \ .$$

On a log-log scale

$$\log(1 - F_s(x)) \sim -ax$$

Pareto (Type I) behaviour

$$1 - F_S(x)\text{Prob}\{S > x\} = \sim (\frac{S}{S_0})^{-a}$$

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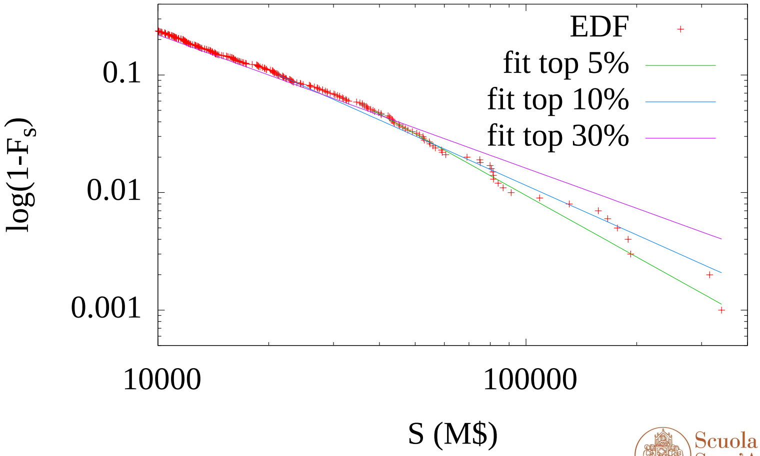
Fortune 500, year 2006



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Estimated power Law

sample size	5%	10%	30%
$\hat{a}$	1.74	1.39	1.13
$\hat{S}_0/10^4$	0.68	0.41	0.26

Estimates depend on sample cut-off. They are constrained between 1 and 2.

The Theoretical Framework

The Islands Model

- The market consists of a number of independent submarkets (islands)
- Each market is large enough to support exactly one plant
- There exists a set of preexisting business opportunities or equivalently there's a constant arrival of new opportunities
- These opportunities are independent each other
- Firm's size is measured by the number of opportunities it has taken up

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Herbert Simon

Herbert Simon is the father of the so-called *Empirically Based Industrial Dynamics*.
Gibrat's RW generates the Log-Normal distribution if all the elements in the population starts the "walk" at the same time.

Is that plausible? Simon considers a different stochastic process in which new entrants are an integral part of the process itself.

Economic theory has little to say about the distribution of firm sizes:

1. static cost theory (constant or U-shaped cost curves) provides no predictions
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Simon's model: assumptions

1. There is a minimum size, S_m, of firms in an industry
2. Size has no effect upon the expected percentage growth of a firm
   - empirically observed
   - implied by another empirical fact: constant returns to scale above a certain minimal threshold (Bain).
3. New firms are being born in the smallest size-class at a constant rate

Under these assumptions the steady-state distribution of the process is:

$$f(S) = \rho B(S, \rho + 1) \qquad \text{Yule distribution} \tag{3}$$

and

$$\lim_{S\to\infty} f(S) = \rho\Gamma(\rho + 1)S^{-(\rho+1)} \qquad \text{Pareto tail} \tag{4}$$

Remark: the entry process is crucial!

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- The Yule distribution (but also the Log-Normal) generally fits the data quite well
- The observed frequencies are Pareto in the upper tail: Yule distribution is OK not the Log-Normal
- Gibrat's Law seems verified by data. The story is not simple:
  1. Weak and strong Gibrat's law
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Simon's model: Implications for Economic Policy

Concentration ratio: when one fits a distribution function to observed data on the basis of a theoretical model it is reasonable to ground his measure of concentration on the parameters of the distribution function.

In the Simon model there is only one parameter ρ

$$\rho = \frac{1}{1 - \frac{G_N}{G}}$$

G_N is the share of growth of new firms: the same equilibrium distribution can be obtained with various degrees of mixing, i.e. with various amounts of firm mobility among size classes.

Simon Revisited

Critique on the robustness of any empirical regularity on size distribution: “All families of distributions tried so far fail to describe at least some industry well”(Schmalansee, 1989)

John Sutton considers the theoretical framework developed by Simon reversing the question: can we put any restrictions on the shape of the size distribution?

Rejection of the Gibrat’s Law in favour of a weaker hypothesis: the probability that the next market opportunity is taken by any currently active firm is non-decreasing in the size of that firm.

Under these assumption a lower bound to concentration is derived and used to empirically validate the model.

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$$s_{ij}(t) = \log(S_{ij}(t)) - < \log(S_{ij}(t)) >_i \tag{5}$$

Main results on empirical firms size densities

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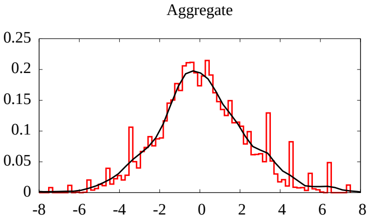
Empirical Size Densities - US



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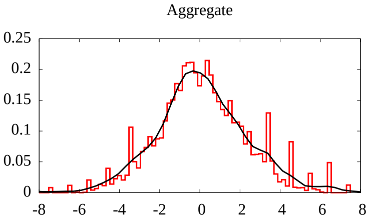

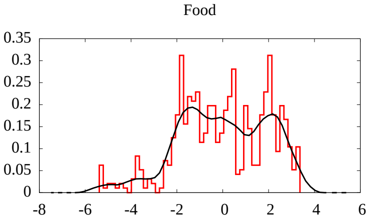

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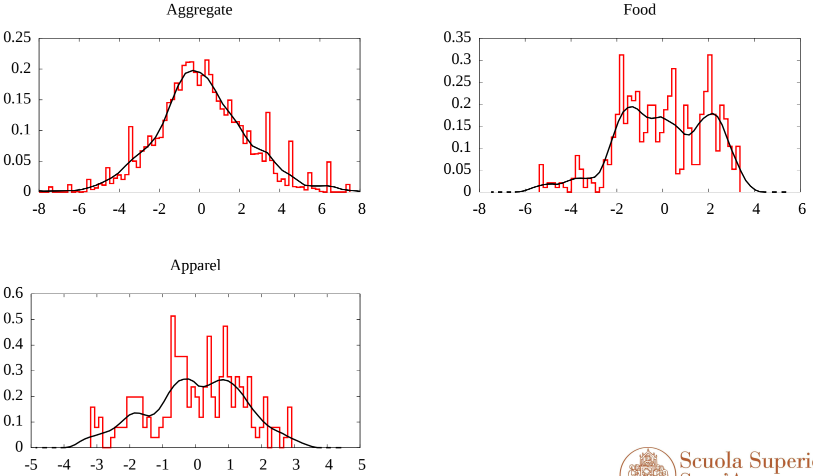

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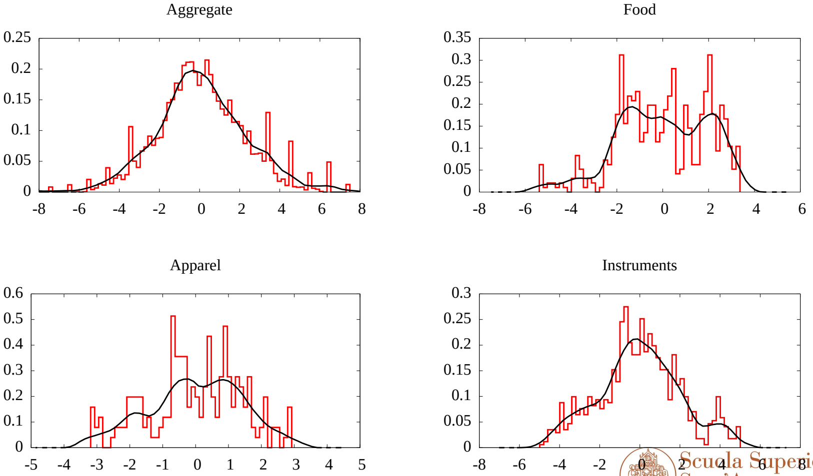

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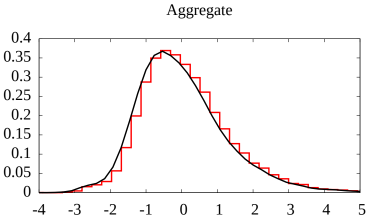

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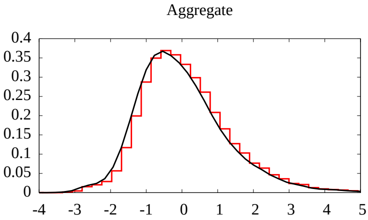

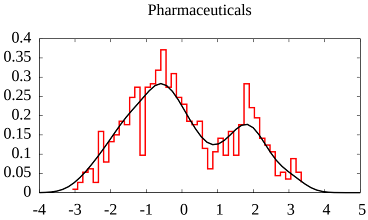

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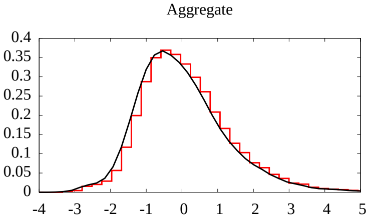

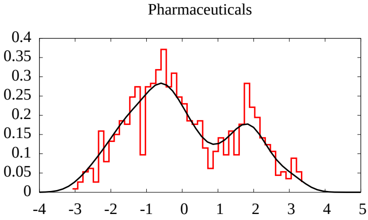

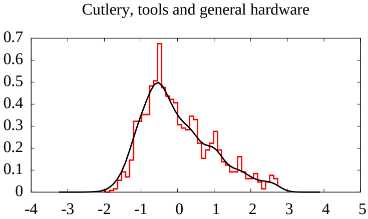

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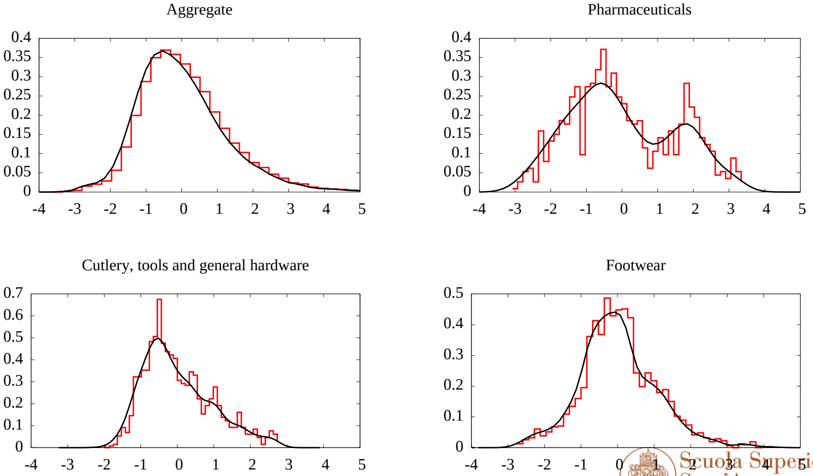

Firms Growth Rates

We build firms growth rates as the first difference of S_{ij}

$$g_{ij}(t) = s_{ij}(t) - s_{ij}(t-1) \tag{6}$$

Main results on empirical growth rates densities

1. shape is stable over time
2. display similar shapes across sectors
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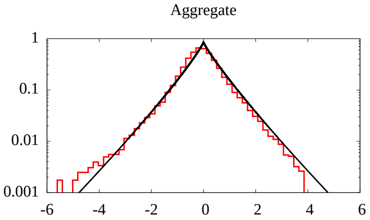

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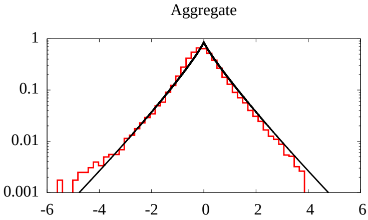

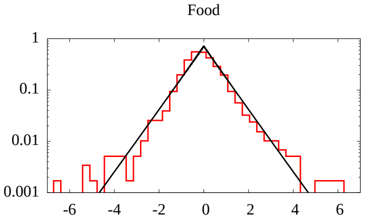

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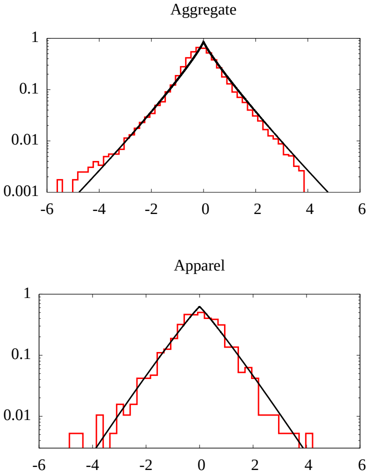

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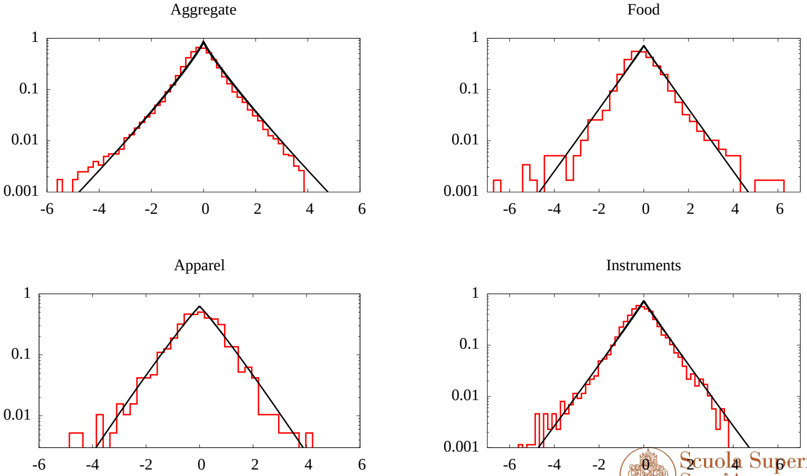

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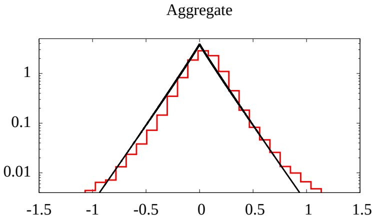

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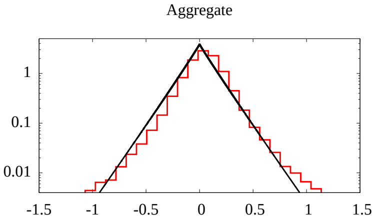

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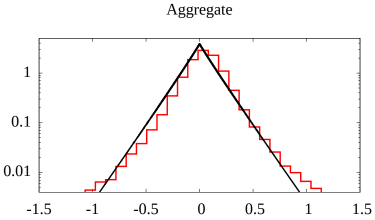

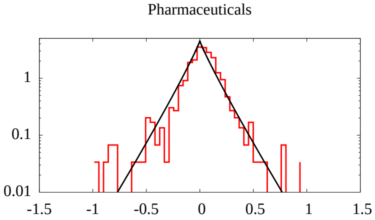

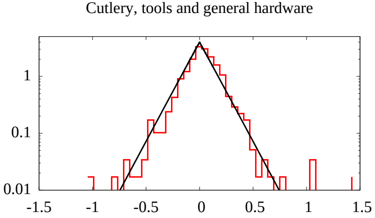

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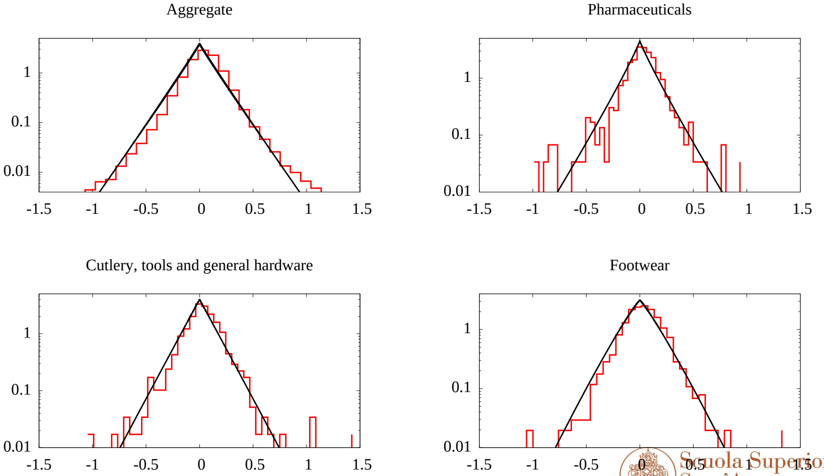

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The Subbotin Distribution

$$f_S(x) = \frac{1}{2ab^{1/b}\Gamma(1/b+1)}\, e^{-\frac{1}{b}\left|\frac{x-\mu}{a}\right|^b} \tag{7}$$

Mikhail Fyodorovich Subbotin (1883-1966)

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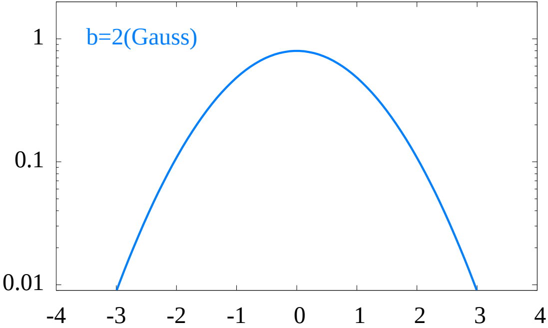

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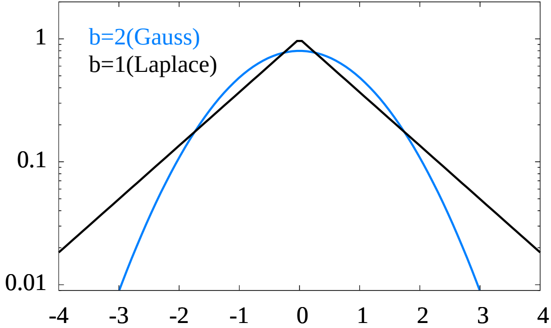

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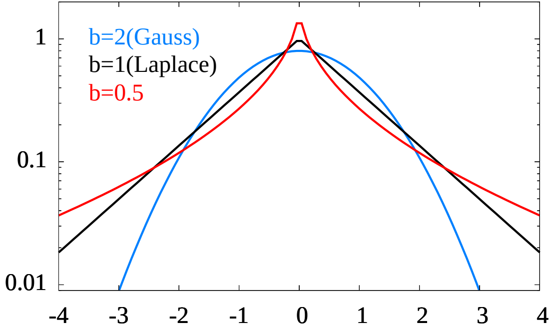

ML Estimation Procedure

We consider:

$$
\begin{aligned}
&-\log(L_S(x; a, b, \mu)) = \\
&\qquad n \log \left(2b^{1/b}\, a\, \Gamma(1 + 1/b)\right) + (ba^b)^{-1} \sum_{i=1}^{n} |x_i - \mu|^b \qquad (8)
\end{aligned}
$$

and we minimize it with respect to the parameters using a multi-step procedure.

These ML estimators are asymptotically consistent in all the parameter space, asymptotically normal for $b > 1$ and asymptotically efficient for $b > 2$.

Estimates on Italian Sectors

		Parameter b		Parameter a	
Ateco code	**Sector**	**Coef.**	**Std Err.**	**Coef.**	**Std Err.**
151	Production, processing and preserving of meat	0.83	0.05	0.089	0.004
155	Dairy products	0.91	0.07	0.080	0.004
158	Production of other foodstuffs (brad, sugar, etc...)	0.89	0.05	0.097	0.004
159	Production of beverages (alcoholic and not)	0.88	0.06	0.108	0.006
171	Preparation and spinning of textiles	1.19	0.07	0.142	0.005
172	Textiles weaving	1.12	0.06	0.122	0.004
173	Finishing of textiles	1.11	0.06	0.107	0.004
175	Carpets, rugs and other textiles	1.02	0.08	0.118	0.006
177	Knitted and crocheted articles	0.97	0.05	0.124	0.005
182	Wearing apparel	0.92	0.03	0.120	0.003
191	Tanning and dressing of leather	1.12	0.09	0.140	0.007
193	Footwear	1.12	0.05	0.150	0.004
202	Production of plywood and panels	0.98	0.09	0.104	0.007
203	Wood products for construction	0.94	0.08	0.105	0.007
205	Production of other wood products (cork, straw, etc...)	1.31	0.13	0.106	0.006

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Estimates on US Sectors

		Parameter b		Parameter a	
Ateco code	**Sector**	**Coef.**	**Std Err.**	**Coef.**	**Std Err.**
20	Food and kindred products	0.9888	0.0010	0.7039	0.0005
23	Apparel and other textile products	1.0819	0.0027	0.7664	0.0013
26	Paper and allied products	1.0999	0.0024	0.7663	0.0011
27	Printing and publishing	0.9621	0.0015	0.7115	0.0008
28	Chemicals and allied products	1.0164	0.0004	0.7562	0.0002
29	Petroleum and coal products	1.1841	0.0043	0.8370	0.0019
30	Rubber and miscellaneous plastics products	0.9487	0.0018	0.7148	0.0010
32	Stone, clay, glass, and concrete products	1.1023	0.0039	0.7720	0.0018
33	Primary metal industries	1.1254	0.0015	0.7870	0.0007
34	Fabricated metal products	0.9081	0.0013	0.6639	0.0007
35	Industrial machinery and equipment	0.9466	0.0003	0.6761	0.0002
36	Electrical and electronic equipment	0.8989	0.0003	0.6303	0.0001
37	Transportation equipment	1.0033	0.0011	0.7107	0.0005
38	Instruments and related products	0.9722	0.0004	0.6980	0.0002
39	Miscellaneous manufacturing industries	1.0232	0.0022	0.7447	0.0011

Outline

The Theoretical Framework

Observed growth as the cumulative effect of diverse "events"

$$g(t;T) = s(t+T) - s(t) = \epsilon_1(t) + \epsilon_2(t) + \ldots = \sum_{j=1}^{G(t;T)} \epsilon_j(t)$$

- The Gibrat Tradition: ϵ_j are r.v. independent from size s (strong form: ϵ_j are i.i.d.) Limitation: No interaction among firms
- The "Islands" Models: Simon introduces Finite number of M opportunities progressively captured by N firms. $G(t;T)$ becomes a r.v. Limitation: Equipartition of opportunities among firms → Gaussian growth rates

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The Model

Multi-step simulation model

Business Events $\rightarrow$ Micro-Shocks $\rightarrow$ Growth

Self-reinforcing effect in events assignment. Idea of "competition among objects whose *market success*...[is] cumulative or self-reinforcing" (B.W. Arthur)

Discrete time stochastic growth process; at each round a two steps procedure is implemented:

- determine the number of events captured by a firm, $G(t;T)$
- disclose $\epsilon_j \quad j = \{1, \ldots, G(t;T)\}$, i.e. the effect of these events on firm size

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STEP 1 - The Assignment of Business Events

1. Consider an urn with N different balls, each representing a firm
2. Draw a ball and replace with TWO of the same kind. (Here the first draw from an urn with two types of ball)

3. Repeat this procedure M times

RESULT: partition of M events on N firms.

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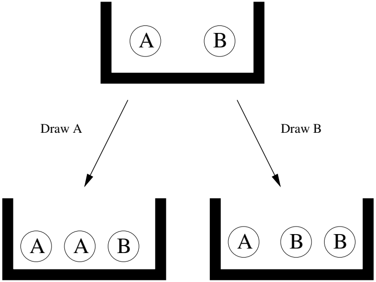

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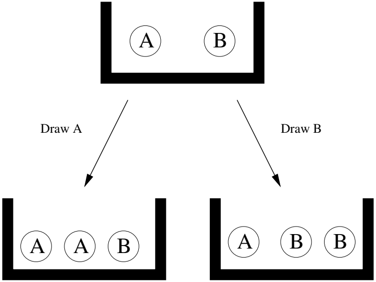

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STEP 2 - The Generation of Shocks

From the previous assignment procedure

$$m_i(t) = \text{\# of opportunity given to firm i at time t}$$

A very simple relation between "opportunities" and growth:

$$s_i(t+T) - s_i(t) = \sum_{j=1}^{m_i(t)+1} \epsilon_j(t) \tag{9}$$

ϵ are i.i.d. with a common distribution $f(\epsilon)$.

Run the simulation and collect statistics.

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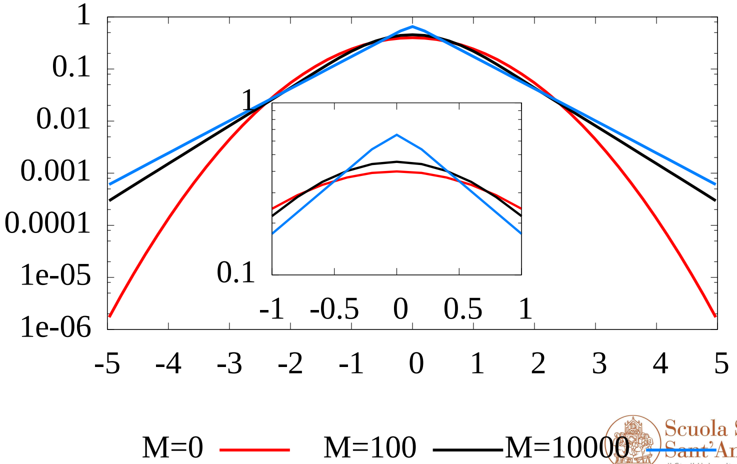
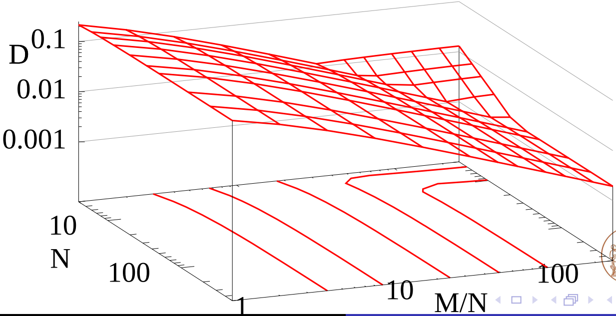
Simulation Results

Growth rates densities for $N = 100$ and different values of M.



Simulation Results - Cont'd

We define $D = |F_{\text{model}}(x; M, N) - F_{\text{L}}(x)|$ the absolute deviation between the empirical growth rates distribution (as approximated by the Laplace) and the distribution predicted by the model. Here D as a function of the number of firms N and the average number of micro-shocks per firm M/N.



Why does the Model work?

The unconditional growth rates distribution implied by this model is given by

$$\sum_{h=0}^{M} \underbrace{P(h;N,M)}_{\text{Events Distribution}} \underbrace{F(x;v_0)^{\star(h+1)}}_{\text{Distribution of the sum of h micro-shocks}} .$$

In the assignment procedure above P follows a Bose-Einstein

$$P(h;N,M) = \frac{P(X)}{P(X|m_1 = h)} = \frac{\binom{N+M-h-2}{N-2}}{\binom{N+M-1}{N-1}}$$

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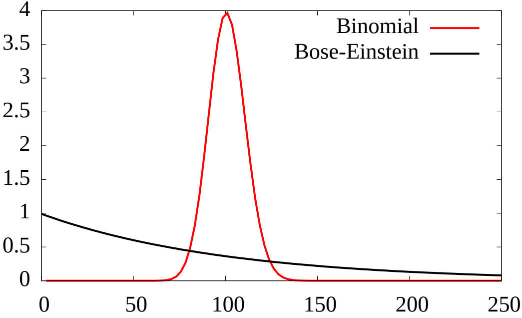
Occupancy Statistics



Bose-Einstein and binomial with $N = 100$ and $M = 10,000$.

"Large Industry" Limit

Theorem

Suppose that the micro-shocks distribution possesses the second central moment $\sigma_\epsilon^2 < \infty$. Under the Polya opportunities assignment procedure the firms growth rates distribution converges in the limit for $N, M \to \infty$ to a Laplace distribution with parameter $\sqrt{v/2}$, i.e.

$$\lim_{M,N\to\infty} f_{\text{model}} = f_L(x; \sqrt{v/2}) = \frac{1}{\sqrt{2v}} e^{-\sqrt{2/v}\,|x|}$$

where $v = \sigma_\epsilon^2 M/N$.

Concluding Remarks on the Model

- A new stylized fact has been presented
- We show its robustness under disaggregation
- Our original explanation is based on a general mechanism of short-horizon "dynamic increasing returns" in a competitive environment
- We provide a "Large Industry" Limit Theorem
- Simulations show that "Large" is not so large

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Selected reference:

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Stochastic processes for dummies

Random Walk: consider discrete equally spaces time-intervals, an object wanders on an infinite straight line taking at each time a step leftward or rightward with probability p and $1 - p$ respectively.

The RW considered by Kapteyn and Gibrat is slightly more complicated:

- it in logs, whence the term *geometric*
- the size of the step taken is itself a random variable.

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Stochastic processes for dummies 2

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$$\text{Prob}\left\{s_2, t+T \middle| s_1, t\right\} = f\left\{s_2 - s_1; T\right\} \ .$$

If the process happens in continuous time, it can be described using the notion of *Wiener process* by the Ito equation

$$ds_t = \mu\, dt + \sigma^2 dW \ .$$

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The Databases

FORTUNE 500 Annual ranking of America's largest public corporations as measured by their gross revenue compiled by *Fortune* magazine.

COMPUSTAT U.S. publicly traded firms in the Manufacturing Industry (SIC code ranges between 2000-3999) in the time window 1982-2001. We have 1025 firms in 15 different two digit sectors.

MICRO.1 Developed by the Italian Statistical Office(ISTAT). More than 8000 firms with 20 or more employees in 97 sectors (3-digit ATECO) in the time window 1989-1996. We use 55 sectors with >44 firms.

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2. Finding simple generalizations that describe the facts to some degree of approximation
3. Finding Limiting conditions under which the deviations of facts from generalization might be expected to decrease
4. Explaining why the generalization “should” fit the facts
5. The explanatory theories generally make predictions that go beyond the simple generalizations and hence suggest new empirical tests.

Scuola Superiore Sant'Anna
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