## Dynamics and time series: theory and applications

Stefano Marmi Scuola Normale Superiore Lecture 7, Feb 10, 2010 Il corso si propone di fornire un'introduzione allo studio delle applicazioni dei sistemi dinamici allo studio delle serie temporali e al loro impiego nella modellizzazione matematica, con una enfasi particolare sull'analisi delle serie storiche economiche e finanziarie. Gli argomenti e i problemi trattati includeranno (si veda la pagina web del docente <u>http://homepage.sns.it/marmi/</u>):

Introduzione ai sistemi dinamici e alle serie temporali. Stati stazionari, moti periodici e quasi periodici. Ergodicità, distribuzione uniforme delle orbite. Tempi di ritorno, diseguaglianza di Kac. Mescolamento. Entropia di Shannon. Entropia di Kolmogorov-Sinai. Esponenti di Lyapunov. Entropia ed elementi di teoria dell'informazione. Catene di Markov. Scommesse, giorchi probabilistici, gestione del rischio e criterio di Kelly.

Introduzione ai mercati finanziari: azioni, obbligazioni, indici. Passeggiate aleatorie, moto browniano geometrico. Stazionarietà delle serie temporali finanziarie. Correlazione e autocorrelazione. Modelli auto regressivi. Volatilità, eteroschedasticità ARCH e GARCH. L'ipotesi dei mercati efficienti. Arbitraggio. Teoria del portafoglio e il Capital Asset Pricing Model.

Modalità dell'esame: Prova orale e seminari

### Sistemi dinamici e teoria dell'informazione:

Benjamin Weiss: "Single Orbit Dynamics", AMS 2000

Thomas Cover, Joy Thomas "Elements of Information Theory" 2nd edition, Wiley 2006

#### Serie temporali:

Holger Kantz and Thomas Schreiber: *Nonlinear Time Series Analysis*, Cambridge University Press 2004 Michael Small: *Applied Nonlinear Time Series Analysis*. *Applications in Physics, Physiology and Finance*, World Scientific 2005

#### Modelli matematici in finanza e analisi delle serie storiche:

M. Yor (Editor): Aspects of Mathematical Finance, Springer 2008

John Campbell, Andrew Lo and Craig MacKinlay: The Econometrics of Financial Markets, Princeton University Press, 1997

Thomas Bjork: *Arbitrage Theory in Continuous Time* (Oxford Finance) Stephen Taylor: "*Modelling Financial Time Series*" World Scientific 2008 Keith Cuthbertson, Dirk Nitzsche "*Quantitative Financial Economics*" John Wiley and Sons (2004)

- Lecture 1: An introduction to dynamical systems and to time series. (Today, 2 pm 4 pm Aula Dini)
- Lecture 2: Ergodicity. Uniform distribution of orbits. Return times. Kac inequality Mixing (Thu Jan 14, 2 pm 4 pm Aula Fermi) by Giulio Tiozzo
- Lecture 3: Kolmogorov-Sinai entropy. Randomness and deterministic chaos. (Wen Jan 20, 2 pm 4 pm Aula Bianchi) by Giulio Tiozzo
- Lecture 4: Introduction to financial markets and to financial time series (Thu Jan 21, 2 pm 4 pm Aula Bianchi Lettere)
- Lecture 5: Central limit theorems (Wen Jan 27, 2 pm 4 pm Bianchi) by Giulio Tiozzo
- Lecture 6: Financial time series: stylized facts and models (Thu Jan 28, 2 pm 4 pm Bianchi)
- Lecture 7: The Efficient Market Hypothesis (Wen Feb 10)
- Lecture 8: An introduction to market microstructure and to high frequency finance, by Fabrizio Lillo (Thu Feb 11, Aula Dini)
- Lecture 9 on Wen Feb 17 No lecture on Thu Feb 18
- Lectures 10 and 11 (including a seminar by A. Carollo) Wen Feb 24 and Thu Feb 25
- Lectures 12 and 13 Wen Mar 3 and Thu Mar 4
   Feb 10, 2010
   S. Marmi Dynamics and time series -Lecture 7: Efficient markets hypothesis

- Seminar I: An introduction to high frequency finance and to market microstructure (Fabrizio Lillo, Palermo, Thu Feb11)
- Seminar II: TBA (Angelo Carollo, Palermo, Thu Feb 25)
- Seminar III: TBA (Massimiliano Marcellino, European University Institute, Thu Mar 18)

- Challenges and experiments:
  - 0. blog?
  - 1. statistical arbitrage in sports betting: collecting time series, etc..
  - 2. nonstationarity and volatility of financial series

Today's bibliography:

S.J. Taylor "Asset price dynamics, volatility and prediction" Princeton University Press, Ch. 5

K. Cuthbertson "Quantitative Financial Economics" Chapter 4 Valuation and 5 The efficient markets hypothesis

Eugene Fama: "Foundations of Finance" Chapter 5: Efficient Capital Markets

Elroy Dimson and Massoud Mussavian: "A brief history of market efficiency" European Financial Management, Volume 4, Number 1, March 1998, pp 91-193

M. Yor (Editor): Aspects of Mathematical Finance, Springer 2008, especially chapters 1 "Financial Uncertainty, Risk Measures and Robust Preferences" by H. Föllmer and chapter 2 "The Notion of Arbitrage and Free Lunch in Mathematical Finance" by W. Schachermayer

# Reminder on the fundamentals of investing

- Investment returns are strongly related to their risk level
- Usually and loosely risk is quantified using volatility (standard deviation)
- <u>Government (U.S., European, etc) bills /bonds (short/long term bonds 1month-1year / 2-30 years )</u>. Bonds are also issued by companies to finance their operations
- <u>T.I.P.</u>: inflation indexed bonds which guarantee a positive real return
- <u>Stocks</u>: risky but higher returns. Each share represents a given fraction of the ownership of the company.
- Certain stocks pay *dividends*, cash payments reflecting profits returned to shareholders. Other stocks reinvest all returns back into the business.
- *Exchanges:* places where buyers/sellers trade *securities* (stocks, bonds, options, futures, commodities). They provide *liquidity*, i.e.ability to buy and sell securities quickly, inexpensively, and at fair market value.
- *Commodities* are types of goods which can be defined so that they are largely indistinguishable in terms of quality (e.g. orange juice, gold, cotton, pork bellies). Commodities markets exist to trade such products, from before they are produced to the moment of shipping. The existence of agricultural futures gives suppliers and consumers ways to protect themselves from unexpected changes in prices.
- •Currency Markets: The largest financial markets by volume trade different types of currency, such as dollars, Euros, and Yen.
- •The *spot price* gives the cost of buying a good now, while *futures* permit one to buy the right to buy or sell goods at fixed prices at some future date.

Suggested web lectures:

1. Robert Shiller (Yale): Efficient markets vs. excess volatility (Lecture 6 of 26 from the course <u>Financial Markets</u>)

http://academicearth.org/lectures/efficient-markets-excess-volatlity Several theories in finance relate to stock price analysis and prediction. The efficient markets hypothesis states that stock prices for publicly-traded companies reflect all available information. Prices adjust to new information instantaneously, so it is impossible to "beat the market." Furthermore, the random walk theory asserts that changes in stock prices arise only from unanticipated new information, and so it is impossible to predict the direction of stock prices. Using statistical tools, we can attempt to test the hypotheses and to predict future stock prices. These tests show that efficient markets theory is a half-truth: it is difficult but not impossible for some people to beat the market.

### 2. http://academicearth.org/lectures/andrew-redleaf

Andrew Redleaf, a Yale graduate and manager of Whitebox Advisors, a hedge fund, discusses his experience with financial markets. He addresses one of the fundamental questions in finance--whether or not markets are efficient--and concludes that although they don't seem to be efficient, beating the market is very difficult. Mr. Redleaf discusses his thoughts about psychological barriers that make markets inefficient. He also comments on his beliefs regarding risk management and how people are compensated for mitigating risks, rather than for taking on risk as is often perceived. He ends by answering several questions from students.

## Arbitrage

*Arbitrage* is a trading strategy which takes advantage of two or more securities being inconsistently priced relative to each other. In financial and economics theory arbitrage is the practice of taking advantage of a price differential between two or more markets or assets: striking a combination of matching deals that capitalize upon the imbalance, the profit being the difference between the prices. When used by academics, an arbitrage is a transaction that involves no negative cash flow at any probabilistic or temporal state and a positive cash flow in at least one state; in simple terms, a risk-free profit. Advanced arbitrage techniques involve sophisticated mathematical analysis and rapid trading.

## Arbitrage and market efficiency

Arbitrage is a trading strategy which takes advantage of two or more securities being inconsistently priced relative to each other. An arbitrage opportunity is the possibility to make a profit in a financial market without risk and without net investment of capital. An arbitrage is a transaction that involves no negative cash flow at any probabilistic or temporal state and a positive cash flow in at least one state; in simple terms, a risk-free profit.

The principle of no arbitrage states that a mathematical model of a financial market should not allow for arbitrage possibilities.

This principle is fundamental and can be used for calculating the price of financial assets.

To apply this principle to less trivial cases, we consider a – still extremely simple – mathematical model of a financial market: there are two assets, called the bond and the stock. The bond is riskless, hence – by definition – we know what it is worth tomorrow. For (mainly notational) simplicity we neglect interest rates and assume that the price of a bond equals  $\in$ 1 today as well as tomorrow, i.e.

$$B_0 = B_1 = 1. (1)$$

The more interesting feature of the model is the stock which is risky: we know its value today, say  $S_0 = 1$ , but we do not know its value tomorrow. We model this uncertainty stochastically by defining  $S_1$  to be a random variable depending on the random element  $\omega \in \Omega$ . To keep things as simple as possible, we let  $\Omega$  consist of two elements only, g for 'good' and b for 'bad', with probability  $\mathbf{P}[g] = \mathbf{P}[b] = \frac{1}{2}$ . We define  $S_1(\omega)$  to equal 2 or  $\frac{1}{2}$  according to whether

$$S_1(\omega) = \begin{cases} 2 & \text{for } \omega = g \\ \frac{1}{2} & \text{for } \omega = b. \end{cases}$$
(2)

Now we introduce a third financial instrument in our model, an *option on the stock* with strike price K: the buyer of the option has the right – but not the obligation – to buy one stock at time t = 1 at the predefined price K. To fix ideas let K = 1. A moment's reflexion reveals that the price  $C_1$  of the option at time t = 1 (where Cstands for *contingent* claim) equals

$$C_1 = (S_1 - K)_+,$$
 (3)

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i.e. in our simple example

$$C_1(\omega) = \begin{cases} 1 & \text{for } \omega = g \\ 0 & \text{for } \omega = b. \end{cases}$$
(4)

Hence we know the value of the option at time t = 1, contingent on the value of the stock. But what is the price of the option today?

At this stage the reader might consult the financial section of a newspaper or the web to see some 'life' examples on quoted option prices.

The classical approach, used by actuaries for centuries, is to price contingent claims by taking expectations, which leads to the value  $C_0 := \mathbf{E}[C_1] = \frac{1}{2}$  in our example. Although this simple approach is very successful in many actuarial applications, it is not at all satisfactory in the present context. Indeed, the rationale behind taking the expected value as the price of a contingent claim is the following: in the long run the buyer of an option will neither gain nor lose on average. We rephrase this fact in a financial lingo: the performance of an investment in the option would on average equal the performance of the bond. However, a basic feature of finance is that an investment into a risky asset should, on average, yield a better performance than an investment in the bond (for the skeptical reader: at the least these two values should not necessarily coincide). In our 'toy example' we have chosen the numbers such that  $\mathbf{E}[S_1] = 1.25 > 1 = \mathbf{E}[B_1]$ , so that on average the stock performs better than the bond.

The Notion of Arbitrage and Free Lounghein Mathematical Finance W. Schachermayer Lecture 7: Efficient markets hypothesis

### 3 Pricing by No Arbitrage

The Notion of Arbitrage and Free Lunch in Mathematical Finance W. Schachermayer

A different approach to the pricing of the option goes like this: we can buy at time t = 0 a *portfolio* consisting of  $\frac{2}{3}$  of stock and  $-\frac{1}{3}$  of bond. The reader might be puzzled about the negative sign: investing a negative amount in a bond – 'going short' in financial lingo – means to borrow money.

One verifies that the value  $\Pi_1$  of the portfolio at time t = 1 equals 1 or 0 depending on whether  $\omega$  equals g or b. The portfolio 'replicates' the option, i.e.

$$C_1 \equiv \Pi_1. \tag{5}$$

We are confident that the reader now sees why we have chosen the above weights  $\frac{2}{3}$  and  $-\frac{1}{3}$ : the mathematical complexity of determining these weights such that (5) holds true amounts to solving two linear equations in two variables.

The portfolio  $\Pi$  has a well-defined price at time t = 0, namely  $\Pi_0 = \frac{2}{3}S_0 - \frac{1}{3}B_0 = \frac{1}{3}$ . Now comes the 'pricing by no arbitrage' argument: equality (5) implies that we also must have

$$C_0 = \Pi_0$$
 (6)

whence  $C_0 = \frac{1}{3}$ . Indeed, suppose that (6) does not hold true; to fix ideas, suppose we have  $C_0 = \frac{1}{2}$  as above. This would allow an arbitrage by buying ('going long in') the

portfolio  $\Pi$  and simultaneously selling ('going short in') the option *C*. The difference  $C_0 - \Pi_0 = \frac{1}{6}$  remains as arbitrage profit at time t = 0, while at time t = 1 the two positions cancel out *independently of whether the random element*  $\omega$  equals g or b. Feb 10, 2010 Lecture 7: Efficient markets hypothesis

## What is an efficent capital market?

An efficient capital market is a market which is efficient in processing information: the prices of securities observed at any time are based on "correct" evaluation of all information available at that time. Prices "fully reflect" available information. The prices are always "fair", they are good indicators of value

The concept of market efficiency had been anticipated at the beginning of the century: Bachelier (1900) writes "past, present and even discounted future events are reflected in market price, but often show no apparent relation to price changes". And also "if the market, in effect, does not predict its fluctuations, it does assess them as being more or less likely, and this likelihood can be evaluated mathematically".

## Weak vs. strong efficiency

More formally: a capital market is said to be efficient if it fully and correctly reflects all relevant information in determining security prices. Formally, the market is said to be efficient with respect to some information set,  $\Theta_t$ , if security prices would be unaffected by revealing that information to all participants. Moreover, efficiency with respect to an information set,  $\Theta_t$ , implies that it is impossible to make economic profits by trading on the basis of  $\Theta_t$ .

The weak form of the efficient market hypothesis claims that prices fully reflect the information implicit in the sequence of **past prices**. The semi-strong form of the hypothesis asserts that <u>prices</u> reflect all relevant information that is **publicly** available, while the strong form of market efficiency asserts information that is known to any participant is reflected in market prices.

\*LE MONDE | 07.01.03 | 12h43 \*MIS A JOUR LE 08.01.03 | 13h14

### Point de vue

### Le mythe funeste des marchés efficients, par Jean-Philippe Bouchaud

Une fois de plus, le repli désordonné des Bourses mondiales et ses conséquences préoccupantes sur l'économie dite "réelle" (débâcle financière de géants industriels, baisse auto-entretenue de la confiance, etc.) pose de nombreuses questions sur la nature, le fonctionnement et le rôle, économique et social, des marchés financiers. L'attribution récente – simple coïncidence ou message opportun ? – du prix Nobel d'économie à Daniel Kahnemann et Vernon Smith, deux trouble-fête qui mettent en doute le paradigme dominant, donne une actualité supplémentaire à des questions parfois théoriques, mais qui méritent, me semble-t-il, une plus grande publicité.

L'économie néoclassique présente le marché financier comme un baromètre infaillible de l'activité humaine, dont le rôle cardinal est de donner une valeur précise à tout ce qui peut s'échanger. Le prix de marché est censé représenter la résultante collective des anticipations d'agents parfaitement rationnels. Cette synthèse optimale de l'information, supérieure en qualité à toutes ses parties, permettrait au marché de valoriser correctement et à tout instant un futur économique incertain et de fixer des prix de référence stables qui déterminent nombre de décisions stratégiques. Les prix rationnels devraient, en principe, peu changer au cours du temps, sauf en cas d'imformations no verselles dont l'incidence est majeure. Lecture 7: Efficient markets hypothesis

La théorie des marchés "efficients", parfaitement cohérente du point de vue logique, mais -, on s'en doute - fort peu réaliste, est pourtant le socle théorique de l'ultralibéralisme. Elle est aussi l'une des pierres angulaires de la science économique telle qu'elle est enseignée, depuis plusieurs dizaines d'années, à ceux qui ont et auront la charge de gérer, à des degrés divers, l'économie mondiale. Or les idées recues laissent toujours des traces, surtout lorsqu'elles font partie d'un système intellectuellement séduisant. Dans ce cadre, le marché a toujours raison, puisqu'il se place à un méta-niveau, démiurge inaccessible aux individus. Défier le marché est donc au mieux présompteux, au pis suicidaire.

Cette idée est à la fois confortable et dangereuse. Confortable, comme tout instrument de mesure fiable : le prix du march é tout à la fois guide et justifie les décisions économiques et industrielles. Investir dans les pays émergents dans l'économie Internet ? Oui, puisque les marchés, par les
valorisations astronomiques des sociétés *dot-com*, nous disent que cette nouvelle économie atteindra des niveaux de rentabilité inédits. Racheter
Orange ou d'autres sociétés technologiques à des prix faramineux qui maintenant grèvent de manière inquiétante les comptes de France
Télécom ou de Vivendi ? C'est le prix du marché, indiscutable et tyrannique - qui pourrait en effet se prévaloir de façon crédible d'une clairvoyance
supérieure à celle distillée par le consensus collectif ?

## More critiques to the EMH

Grossman and Stiglitz ("On the Impossibility of Informatioally Efficient Markets, American Economic Review, 70, 393-408, 1980) argue that perfectly informationally efficient markets are an impossibility. Roughly speaking the idea is more or less that if markets were perfectly efficient, there would be no profit to gathering information, in which case (in an equilibrium world) there would be little reason to trade and markets would eventually collapse.

Alternatively, the degree of market inefficiency determines the effort investors are willing to expend to gather and trade on information, hence a non-degenerate market equilibrium will arise only when there are sufficient profit opportunities, i.e., inefficiencies, to compensate investors for the costs of trading and information-gathering. The profits earned by these attentive investors may be viewed as "economic rents" that accrue to those willing to engage in such activities. Who are the providers of these rents? Black (1986) gave us a provocative answer: "noise traders", individuals who trade on what they consider to be information but which is, in fact, merely noise.

(From A. Lo, The Adaptive Market Hypothesis, Journal of Portfolio Management 2004)

### Can Predicable Patterns in Market Returns be Exploited Using Real Money? Not likely.

BURTON G. MALKIEL is Chemical Bank Chairman's Professor of Economics at Princeton University in Princeton, NJ.

Feb 10, 2010 **30TH ANNIVERSARY ISSUE 2004** 

n the occasion of the 30th anniversary of this Journal, which has done so much to enhance the professionalism and the intellectual content of portfolio management, it seems appropriate to examine the shift of belief in the academy as to the efficiency of financial markets and the degree to which stock prices are predictable. In this article, I review some important academic contributions of the past three decades, and present some new empirical evidence concerning stock market predictability.

I argue that it is not realistically possible to earn the excess returns that the academic findings about predictability would seem to suggest. I conclude that considerable skepticism is warranted with respect to active portfolio management strategies as well as strategies designed to alter asset allocations over time on the basis of relative valuations.

#### FROM EFFICIENT MARKETS TO STOCK MARKET PREDICTABILITY

In 1970, when Eugene Fama published his survey, "Efficient Capital Markets: A Review of Theory and Empirical Work," there was broad consensus in the academy that our security markets were extremely efficient. Stock prices were assumed to reflect information without delay. If prices accurately represent the future prospects of each firm, then one stock would be just as attractive as another, and an investor who tries to switch from security to security in an attempt to gain excess returns would be unsuccessful.

The notion of efficiency was associated with the view that stock prices would move unpredictably. If prices change S. Marmi - Dynamics and time series -

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## BENJAMIN GRAHAM

THE MEMOIRS OF THE DEAN OF WALL STREET

### Benjamin Graham (5/8/1894-9/21/1976)

was an American economist and professional investor. First proponent of value investing, an investment approach he began teaching at Columbia Business School in 1928 and subsequently refined with David Dodd through various editions of their famous book *Security Analysis*. His most famous disciples is Warren Buffet, who credits Graham as grounding him with a sound intellectual investment framework. Graham recommended that investors spend time and effort to analyze the financial state of companies. When a company is available on the market at a price which is at a discount to its fair value, a *margin of safety* exists, which makes it suitable for investment.

Graham's favorite allegory is that of Mr. Market, a fellow who turns up every day at the stock holder's door offering to buy or sell his shares a a different price. Often, the price quoted by Mr. Market seems plausible, but often it is ridiculous. The investor is free to either agree with his quoted price and trade with him, or to ignore him completely. Mr. Market doesn't mind this, and will be back the following day to quote another price. The point is that the investor should not regard the whims of Mr. Market as determining the value of the shares that the investor owns. He should profit from market folly rather than participate in it. The investor is best off concentrating on the real life performance of his companies and receiving dividends, rather than being too concerned with Mr. Market soften irrational behavior.



Benjamin Graham & David L. Dodd



Charles Henry Dow (b.11/6/1851, d.12/4/1902) cofounded *Dow Jones & Company* with E. Jones and C. Bergstesser. Dow also founded *The Wall Street Journal*, which became one of the most respected financial publications in the world. He also invented the famous *Dow Jones Industrial Average* as part of his research into market movements. Furthermore he developed a series of principles for understanding and analyzing market behavior which later became known as *Dow theory*, the groundwork for *technical analysis*.

Dow published the Wall Street Journal beginning in 1889. He wrote during a period of generally rising stock prices from the depression lows in the 1870s to the then all time high in 1901. During that period Dow formulated his theory of the stock market. It consisted of two important components: the cyclical nature of the market and in the longer cycle, the "third wave", the need for confirmation between economically different sectors, specifically the industrials and the railroads. "Nothing is more certain that the market has three well-defined movements which fit into each other. The first is the variation due to local causes and the balance of buying and selling at that particular time. The secondary movement covers a period ranging from 10 days to 60 days, averaging probably between 30 and 40 days. The third movement is the great swing covering from four to six years." 20 Lecture 7: Efficient markets hypothesis

## Fundamental vs. technical analysis

Fundamental analysis maintains that markets may misprice a security in the short run but that the "correct" price will eventually be reached. Analyzing financial statements, management and competitive advantages, one can accurately estimate a "fair value" for the stock. Profits can be made by trading the mispriced security and then waiting for the market to recognize its "mistake" and reprice the security. Technical analysis maintains that all information is reflected already in the stock price, so fundamental analysis is a waste of time. Trends 'are your friend' and sentiment changes predate and predict trend changes. Investors' emotional responses to price movements lead to recognizable price chart patterns. Technical analysis does not care what the 'value' of a stock is. Their price predictions are only extrapolations from historical price patterns.

If the markets is efficient the market price of a stock is the best possible estimate of its value. Fundamental analysis is reduced to a process which can verify that the market estimate of the value of the stock is correct. If the market is not perfectly efficient, the price of a stock can differ from its value quite considerably and fundamental analysis may be used profitably.

The efficient market hypothesis does not require that the price is always equal to the value: it is sufficient that valuation mistakes do not obey to any logic, so that they are completely random and uncorrelated so that the probability that a given stock is under/overvalued is the same at all times.

### But...what should be the value of a stock?

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Fischer Sheffey Black (January 11, 1938 – August 30, 1995) was an American economist, best known as one of the authors of the famous Black-Feb 10, 2010 Scholes equation. "However, we might define an efficient market as one in which price is within a factor of 2 of value, i.e. the price is more than half of value and less than twice value. The factor of 2 is arbitrary, of course. Intuitively, though, it seems reasonable to me, in the light of sources of uncertainty about value and the strength of the forces tending to cause price to return to value. By this definition, I think almost all markets are efficient almost all of the time. 'Almost all' means at least 90% "

F. Black, Noise, Journal of Finance (1986) p. 533.

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### Noise

### FISCHER BLACK\*

#### ABSTRACT

The effects of noise on the world, and on our views of the world, are profound. Noise in the sense of a large number of small events is often a causal factor much more powerful than a small number of large events can be. Noise makes trading in financial markets possible, and thus allows us to observe prices for financial assets. Noise causes markets to be somewhat inefficient, but often prevents us from taking advantage of inefficiencies. Noise in the form of uncertainty about future tastes and technology by sector causes business cycles, and makes them highly resistant to improvement through government intervention. Noise in the form of expectations that need not follow rational rules causes inflation to be what it is, at least in the absence of a gold standard or fixed exchange rates. Noise in the form of uncertainty about what relative prices would be with other exchange rates makes us think incorrectly that changes in exchange rates or inflation rates cause changes in trade or investment flows or economic activity. Most generally, noise makes it very difficult to test either practical or academic theories about Feb 10, 2010 way that financial or economicDmarkets work. SWes are forced to act largely in the Lecture 7: Efficient markets hypothesis dark.

## What is the "fair value" of a stock? The Gordon model

### 4.1 THE RATIONAL VALUATION FORMULA (RVF)

### Expected Returns are Constant

One of the simplest assumptions one can make is that expected returns are constant. The expected return is defined as

$$E_{t}R_{t+1} = \frac{E_{t}V_{t+1} - V_{t} + D_{t+1}^{e}}{V_{t}}$$
(4.1)

where  $V_t$  is the value of the stock at the end of time t and  $D_{t+1}$  are dividends paid between t and t + 1.  $E_t$  is the expectations operator based on information at time t or earlier,  $\Omega_t$ . The superscript 'e' is equivalent to  $E_t$ ; it helps to simplify the notation and is used when no ambiguity is likely to arise, that is:

$$D_{t+1}^e \equiv E(D_{t+1}|\Omega_t) \equiv E_t D_{t+1}$$

Assume investors are willing to hold the stock as long as it is expected to earn a constant return (= k). We can think of this 'required return' k as that rate of return that is just sufficient to compensate investors for the inherent riskiness of the stock:

KebGutbbortson "Quantitative Financia PEconomics" e series -Lecture 7: Efficient markets hypothesis

$$E_t R_{t+1} = k \quad k > 0 \tag{4.2}$$

The form of (4.2) is known as the *fair game* property of excess returns (see Chapter 5). The stochastic behaviour of  $R_{t+1} - k$  is such that no abnormal returns are made, on *average*: the expected (conditional) excess return on the stock is zero:

$$E_t(R_{t+1} - k | \Omega_t) = 0 (4.3)$$

Using (4.1) and (4.2) we obtain a differential (Euler) equation which determines the movement in 'value' over time:

$$V_t = \delta E_t (V_{t+1} + D_{t+1}) \tag{4.4}$$

where  $\delta = \text{discount factor} = 1/(1+k)$  with  $0 < \delta < 1$ . Leading (4.4) one period:

$$V_{t+1} = \delta E_{t+1} (V_{t+2} + D_{t+1})$$
(4.5)

Now take expectations of (4.5) assuming information is only available up to time t:

$$E_t V_{t+1} = \delta E_t (V_{t+2} + D_{t+2}) \tag{4.6}$$

In deriving (4.6) we have used the *law of iterated expectations*:

$$E_t(E_{t+1}V_{t+2}) = E_t V_{t+2} \tag{4.7}$$

KebGutbbortson "Quantitative Financia PEconomics" e series -Lecture 7: Efficient markets hypothesis in the future. Strictly (4.7) is an assumption that agents use *rational expectations* (see Chapter 5). Equation (4.6) holds for all periods so that

$$E_{I}V_{I+2} = \delta E_{I}(V_{I+3} + D_{I+3}), \text{ etc.}$$
 (4.8)

By successive substitution

$$V_{t} = \delta D_{t+1}^{e} + \delta^{2} D_{t+2}^{e} + \delta^{3} D_{t+3}^{e} + \dots + \delta^{N} (D_{t+N}^{e} + V_{t+N}^{e})$$
(4.9)

Now let  $N \to \infty$  and hence  $\delta^N \to 0$ . If the expected growth in D is not explosive so that  $D_{t+N}^e$  is finite and if  $V_{t+N}^e$  is also finite then:

$$\lim_{n \to \infty} E_t [\delta^N D_{t+N} + V_{t+N}] \to 0 \tag{4.10}$$

Equation (4.10) is known as a terminal condition or *transversality condition* and it rules out rational speculative bubbles (see Chapter 7). Equation (4.9) then becomes:

$$V_t = \sum_{i=1}^{\infty} \delta^i D_{t+i}^e \tag{4.11}$$

Where  $\delta = 1/(1+k)$  we have derived (4.11) under the assumptions:

- expected returns are constant K. Cuthbertson "Quantitative Financial Economics"
- the law of iterated expectations (i.e. RE) holds for all investors
- dividend growth is not explosive and the terminal condition holds
- •Feb 10, 2010 expectations<sup>Lecture 7:</sup> Efficient markets hypothesis

#### Expected Dividend Growth is Constant

A time series model in which (real) dividends grow at a constant rate g is the AR(1) model:

$$D_{t+1} = (1+g)D_t + w_{t+1} \tag{4.16}$$

where  $w_t$  is white noise and  $E(w_{t+1}|\Omega_t) = 0$ . Expected dividend growth from (4.16) is easily seen to be equal to g.

$$E_t D_{t+1} = (1+g)D_t$$
  
(E\_t D\_{t+1} - D\_t)/D\_t = g (4.17)

Note that if the *logarithm* of dividends follows a random walk with drift parameter = g, then this also gives a constant expected growth rate for dividends (i.e.  $\ln D_{t+1} = g + \ln D_t + w_t$ ). The optimal forecasts of future dividends may be found by leading (4.16) one period

$$D_{t+2} = (1+g)D_{t+1} + w_{t+2}$$

and using

$$E_t D_{t+2} = (1+g)E_t D_{t+1} = (1+g)^2 D_t$$

Hence by repeated substitution:

$$E_t D_{t+j} = (1+g)^j D_t \tag{4.18}$$

KebGutbbortson "Quantitative Financia PEconomics" e series -Lecture 7: Efficient markets hypothesis Substituting the forecast of future dividends from (4.18) in the rational valuation formula gives:

$$V_{t} = \sum_{i=1}^{\infty} \delta^{i} (1+g)^{i} D_{t}$$
(4.19)

which after some simple algebra yields

$$V_{t} = \frac{(1+g)}{(k-g)} D_{t} \quad \text{with } (k-g) > 0 \tag{4.20}$$

K. Cuthbertson "Quantitative Financial Economics"

Thus the fair value of the stock depends:

- on the current dividend D<sub>t</sub>
- on the required return rate k
- on the expected growth rate of the dividends

Formula (4.20) is also known as Gordon formula.
Gordon, Myron J. (1959). "Dividends, Earnings and Stock Prices". *Review of Economics and Statistics* 41: 99–105.

## Weak efficiency

In weak-form efficiency excess returns cannot be made by using investment strategies based on historical prices or other historical financial data: for example it will not be possible to make excess returns by using methods such as technical analysis. A trading strategy incorporating historical data, such as price and volume information, will not systematically outperform a buy-and-hold strategy. It is often said that current prices accurately incorporate all historical information, and that current prices are the best estimate of the value of the investment. Prices will respond to news, but if this news is random then price changes will also be random. Technical analysis will not be profitable.

## Assumptions behind technical analysis

- Price is determined only by supply and demand
- Supply and demand are governed by rational and irrational factors. The market continually and automatically weighs all these factors.
- Stock prices tend to move in trends which persist for an appreciable length of time.
- Changes in trend are caused by shifts in demand and supply and these shifts can be detected in the action of the market itself.

## A Quantitative Approach to Tactical Asset Allocation

Journal of Wealth Management (2007) and 2009 update available at the SSRN preprint database, id1347034

### MEBANE T. FABER

This article examines a very simple quantitative market-timing model. This trend following model is examined in-sample on the U.S. stock market since 1900 before out-of-sample testing across more than twenty other markets. The attempt is not to build an optimization model (indeed, the chosen model is decidedly sub-optimal, as evidenced later in the article), but to build a simple trading model that works in the vast majority of markets. The results suggest that a market timing solution is a risk-reduction technique rather than a return-enhancing one. The approach is then examined in an allocation framework since 1972, including such diverse asset classes as the Standard and Poor's 500 Index (S&P 500), Morgan Stanley Capital International Developed Markets Index (MSCI EAFE), Goldman Sachs Commodity Index (GSCI), National Association of Real Estate Investment Trusts Index (NAREIT), and United States Government 10-Year Treasury Bonds. The empirical results are equity-like returns with bond-like volatility and drawdown, and over thirty consecutive years of positive returns. S. Marmi - Dynamics and time series -Feb 10, 2010

Lecture 7: Efficient markets hypothesis

### BUY RULE

Buy when monthly price > 10-month SMA.

- SELL RULE
- Sell and move to cash when monthly price < 10-month SMA.
- 1. All entry and exit prices are on the day of the signal at the close.
- 2. All data series are total return series including dividends, updated monthly.
- 3. Cash returns are estimated with 90-day commercial paper.
- 4. Taxes, commissions, and slippage are excluded .



#### Exhibit 5: S&P 500 Total Returns vs. Timing Total Returns (1900-2008)

|  | S&P 500  | Timing   |
|--|----------|----------|
| Annualized Return                              | 9.21%    | 10.45%   |
| Volatility                                     | 17.87%   | 12.01%   |
| Sharpe (4%)                                    | 0.29     | 0.54     |
| Maximum drawdown                               | (83.66%) | (50.31%) |
| Best Year                                      | 52.88%   | 52.40%   |
| 5. Marmi - Dynamics and time series Worst Year | (43.86%) | (26.87%) |

Lecture 7: Efficient markets hypothesis

## Strong and semi-strong efficiency

In the semi-strong form of the EMH a trading strategy incorporating current publicly available fundamental information (such as financial statements) and historical price information will not systematically outperform a buy-and-hold strategy. Share prices adjust instantaneously to publicly available new information, and no excess returns can be earned by using that information. Fundamental analysis will not be profitable.

In strong-form efficiency share prices reflect all information, public and private, fundamental and historical, and no one can earn excess returns. Inside information will not be profitable.

## Weak, semi-strong and strong EMH

- Weak EMH. "One can not use *past price and volume values* to craft investment strategies that can reliably out perform the over all market."
- Semi-Strong EMH. "One cannot use *publically available information* to to craft investment strategies that can reliably out perform the over all market."
- **Strong EMH**. "One cannot use *any information --- including material, non-public information ---* to to craft investment strategies that can reliably out perform the over all market."



Year

Robert Shiller's plot of the S&P Composite Real <u>Price-Earnings Ratio</u> and Interest Rates (1871–december 2008), from <u>Irrational Exuberance</u>, 2d ed.<sup>[11]</sup> In the preface to this edition, Shiller warns that "[t]he stock market has not come down to historical levels: the price-earnings ratio as I define it in this book is still, at this writing [2005], in the mid-20s, far higher than the historical average. ... People still place too much confidence in the markets and have too strong a belief that paying attention to the gyrations in their investments will someday make them rich, and so they do not make conservative preparations for possible bad outcomes."

# P/E ratios as a predictor of long term U.S. stocks returns

Price-Earnings ratios as a predictor of twenty-year returns based upon the plot by <u>Robert Shiller</u> (Figure 10.1 Irrational Exuberance, Princeton University Press.). The horizontal axis shows the real price/earnings ratio of the S&P500 index (inflation adjusted price divided by the prior ten-year mean of inflationadjusted earnings). The vertical axis shows the geometric average real annual return on investing in the S&P500 index, reinvesting dividends, and selling ten or twenty years later. Data from different ten/twenty year periods is colorcoded as shown in the key. According to Shiller these plots "confirms that long-term investors—investors who commit their money to an investment for ten full years—did do well when prices were low relative to earnings at the beginning of the ten years. Long-term investors would be well advised, individually, to lower their exposure to the stock market when it is high, as it has been recently, and get into the market when it is low."



