

Dynamics and time series: theory and applications

Stefano Marmi

Scuola Normale Superiore

Lecture 4, Jan 21, 2010

Il corso si propone di fornire un'introduzione allo studio delle applicazioni dei sistemi dinamici allo studio delle serie temporali e al loro impiego nella modellizzazione matematica, con una enfasi particolare sull'analisi delle serie storiche economiche e finanziarie. Gli argomenti e i problemi trattati includeranno (si veda la pagina web del docente <http://homepage.sns.it/marmi/>):

Introduzione ai sistemi dinamici e alle serie temporali. Stati stazionari, moti periodici e quasi periodici. Ergodicità, distribuzione uniforme delle orbite. Tempi di ritorno, disuguaglianza di Kac. Mescolamento. Entropia di Shannon. Entropia di Kolmogorov-Sinai. Esponenti di Lyapunov. Entropia ed elementi di teoria dell'informazione. Catene di Markov. Scommesse, giochi probabilistici, gestione del rischio e criterio di Kelly.

Introduzione ai mercati finanziari: azioni, obbligazioni, indici. Passeggiate aleatorie, moto browniano geometrico. Stazionarietà delle serie temporali finanziarie. Correlazione e autocorrelazione. Modelli auto regressivi. Volatilità, eteroschedasticità ARCH e GARCH. L'ipotesi dei mercati efficienti. Arbitraggio. Teoria del portafoglio e il Capital Asset Pricing Model.

Modalità dell'esame: Prova orale e seminari

Sistemi dinamici e teoria dell'informazione:

Benjamin Weiss: *"Single Orbit Dynamics"*, AMS 2000

Thomas Cover, Joy Thomas *"Elements of Information Theory"* 2nd edition, Wiley 2006

Serie temporali:

Holger Kantz and Thomas Schreiber: *Nonlinear Time Series Analysis*, Cambridge University Press 2004

Michael Small: *Applied Nonlinear Time Series Analysis. Applications in Physics, Physiology and Finance*, World Scientific 2005

Modelli matematici in finanza e analisi delle serie storiche:

M. Yor (Editor): *Aspects of Mathematical Finance*, Springer 2008

John Campbell, Andrew Lo and Craig MacKinlay: *The Econometrics of Financial Markets*, Princeton University Press, 1997

Thomas Bjork: *Arbitrage Theory in Continuous Time* (Oxford Finance)

Stephen Taylor: *"Modelling Financial Time Series"* World Scientific 2008

Keith Cuthbertson, Dirk Nitzsche *"Quantitative Financial Economics"* John Wiley and Sons (2004)

- Lecture 1: An introduction to dynamical systems and to time series. (Today, 2 pm - 4 pm Aula Dini)
- Lecture 2: Ergodicity. Uniform distribution of orbits. Return times. Kac inequality Mixing (Thu Jan 14, 2 pm - 4 pm Aula Fermi) by Giulio Tiozzo
- Lecture 3: Kolmogorov-Sinai entropy. Randomness and deterministic chaos. (Wen Jan 20, 2 pm - 4 pm Aula Bianchi) by Giulio Tiozzo
- Lecture 4: Introduction to financial markets and to financial time series (Thu Jan 21, 2 pm - 4 pm Aula Bianchi **Lettere**)
- Lecture 5: Central limit theorems (Wen Jan 27, 2 pm - 4 pm Bianchi) by Giulio Tiozzo
- Lecture 6: Financial time series: stylized facts and models (Thu Jan 28, 2 pm - 4 pm Bianchi)
- Lecture 7: (**Thu Feb 4**, 2 pm - 4 pm Dini)
- Lectures 8 and 9 (including possibly a seminar) Wen Feb 11 and Thu Feb 12
- Lectures 10 and 11 (including possibly a seminar) Wen Feb 18 and Thu Feb 19
- Lectures 12 and 13 (including possibly a seminar) Wen Feb 25 and Thu Feb 26
- Lectures 14 and 15 (including possibly a seminar) Wen Mar 3 and **Wen Mar 10**

- Seminar I: TBA (Fabrizio Lillo, Palermo, Thu Feb11)
- Seminar II: TBA (Angelo Carollo, Palermo, Thu Feb 25)
- Seminar III: TBA (Massimiliano Marcellino, European University Institute, Thu Mar 18)
- Challenges and experiments:
 - 0. blog?
 - 1. statistical arbitrage in sports betting: collecting time series, etc..
 - 2. nonstationarity and volatility of financial series

Today's and next week's bibliography:

R. Cont “Empirical properties of asset returns: stylized facts and statistical issues” Quantitative Finance 1 (2001) 223–236

<http://www.proba.jussieu.fr/pageperso/ramacont/papers/empirical.pdf>

S.J. Taylor “Asset Price Dynamics, Volatility, and Prediction” Princeton University Press (2005). Chapters 2 and 4

Steven Skiena CSE691 Computational Finance class at Stony Brook: <http://www.cs.sunysb.edu/~skiena/691/>

Recommended video lectures for a very beautiful introduction to financial markets:

<http://oyc.yale.edu/economics/financial-markets/>

Financial Markets with Professor Robert Shiller **About the Course**

Financial institutions are a pillar of civilized society, supporting people in their productive ventures and managing the economic risks they take on. The workings of these institutions are important to comprehend if we are to predict their actions today and their evolution in the coming information age. The course strives to offer understanding of the theory of finance and its relation to the history, strengths and imperfections of such institutions as banking, insurance, securities, futures, and other derivatives markets, and the future of these institutions over the next century. [view class sessions >>](#)

About Professor Robert Shiller

Robert J. Shiller is Arthur M. Okun Professor of Economics at Yale University and a Fellow at the International Center for Finance at the Yale School of Management. Specializing in behavioral finance and real estate, Professor Shiller has published in *Journal of Financial Economics*, *American Economic Review*, *Journal of Finance*, *Wall Street Journal*, and *Financial Times*. His books include *Market Volatility*, *Macro Markets* (for which he won the TIAA-CREF's Paul A. Samuelson Award), *Irrational Exuberance*, and *The New Financial Order: Risk in the Twenty-First Century*.

http://openmedia.yale.edu/cgi-bin/open_yale/media_downloader.cgi?file=/courses/spring08/econ252/download/econ252.zip

Free sources of financial time series

finance.yahoo.com

U.S. and European stocks,
many indices

www.federalreserve.gov/releases/

Currencies, etc.

www.crbtrader.com/

Commodities

www.sgindex.com/

various quantitative indexes

www.djindexes.com/

various indexes

Fundamentals of investing

Investment returns are strongly related to their risk level

Usually and loosely risk is quantified using volatility (standard deviation)

U.S. Treasury bills /bonds (short/long term bonds 1month-1year / 2-30 years): very safe (until now...) and very low/medium yield. Most of the price uncertainty for longer term bonds comes from the effect of inflation

T.I.P. : inflation indexed bonds which guarantee a positive real return

Stocks: risky but higher returns (on the long run...). Companies sell shares of stock to raise capital: they ``go public" by agreeing to sell a certain number of shares on an exchange. Each share represents a given fraction of the ownership of the company.

Certain stocks pay *dividends*, cash payments reflecting profits returned to shareholders. Other stocks reinvest all returns back into the business.

In principle, what people will pay for a stock reflects the health of its current business, future prospects, and expected returns. But the current *price* of a stock is completely determined by what people are willing to pay for it. If there were no differences of opinion as to the value of a stock, there would be no trading.

Analisi dei rendimenti degli indici S&P500, Lehman Long Term Government Bonds, MSCI Europe Australasia Far East, FTSE North American Real Estate Investment Trusts e Goldman Sachs Commodities Index dal 1973 al 2007. Tratto da "The case for multi-asset investing. Combining asset classes to enhance risk/return potential", Jennison Dryden-Prudential Investment disponibile online al link :

[http://www.jennisondryden.com/view/upload?docURL=/WDocs/45FB1E842986A540852573E2006BA8C8/\\$File/JD2065MultipleClass.pdf&docType=pdf](http://www.jennisondryden.com/view/upload?docURL=/WDocs/45FB1E842986A540852573E2006BA8C8/$File/JD2065MultipleClass.pdf&docType=pdf)

Periodo	S&P 500	Lehman Long-Term	MSCI EAFE	FTSE NAREIT	Goldman Sachs3	Portafoglio classico:	Portafoglio AA: 20% S&P500 20% Bonds
1973-2007	total return	Government Bond	total return	Equity Index	Commodities Index	50% S&P500	20% EAFE 20% NAREIT
Rendimento annuale medio	10.97%	8.90%	11.09%	13.16%	10.92%	10.31%	12.22%
deviazione standard	17.23%	11.49%	21.58%	21.58%	24.46%	11.67%	9.36%
anno migliore	37.43%	42.08%	69.94%	47.59%	74.96%	34.17%	29.91%
anno peggiore	-26.74%	-8.73%	-23.19%	-21.40%	-35.75%	-11.55%	-9.35%
% anni positivi	71%	80%	74%	80%	74%	80%	89%

Tavola I

Financial markets

An *exchange* is a place where buyers and sellers trade *securities* such as stocks, bonds, options, futures, and commodities.

Each stock is typically traded on a particular exchange. Each exchange has different rules about the qualifications of companies which can be listed on it. Exchanges also differ in the *rules* by which they match buyers to sellers. The exact trading rules and mechanisms can have a significant impact on the price one gets for a given security.

The strength of an exchange's rules and their enforcement impacts the *confidence* of investors and their willingness to invest.

Exchanges provide *liquidity*, the ability to buy and sell securities quickly, inexpensively, and at fair market value.

In general, the more trading that occurs in a security, the greater its liquidity.

Bonds, Commodities and Currencies

Bond markets trade bonds ("loans") made to governments and companies. Bond prices vary according to the *term* (length of time) of the loan, the interest rate and payment schedule, the financial strength of the borrowing party, and the returns available from other investments.

Commodities are types of goods which can be defined so that they are largely indistinguishable in terms of quality (e.g. orange juice, gold, cotton, pork bellies). Commodities markets exist to trade such products, from before they are produced to the moment of shipping. Agricultural futures sell the right to buy a certain amount of a commodity at a particular price at a particular point in the future. The existence of agricultural futures gives suppliers and consumers ways to protect themselves from unexpected changes in prices.

The prices of agricultural commodities are affected by changes in supply and demand resulting from weather, political, and economic forces.

Currency Markets: The largest financial markets by volume trade different types of currency, such as dollars, Euros, and Yen.

The *spot price* gives the cost of buying a good now, while *futures* permit one to buy the right to buy or sell goods at fixed prices at some future date.

Typically, each seller has a buy and sell price for a given currency, and makes their money from the *spread* between these two prices.

Ideally the demand for buying equals selling, or else the prices must change.

Currency markets are used to (a) acquire funds for international trade, (b) hedge against risks of currency fluctuations, (c) speculate on future events.

Stock prices and indices

Stock indices are typically weighted averages of the prices of the component stocks. Usually the weights are proportional to the market capitalization = (price of a share) * (number of existing shares) of the stock.

The same formulae as before are used to calculate returns from index levels. Very often dividends are excluded from the index.

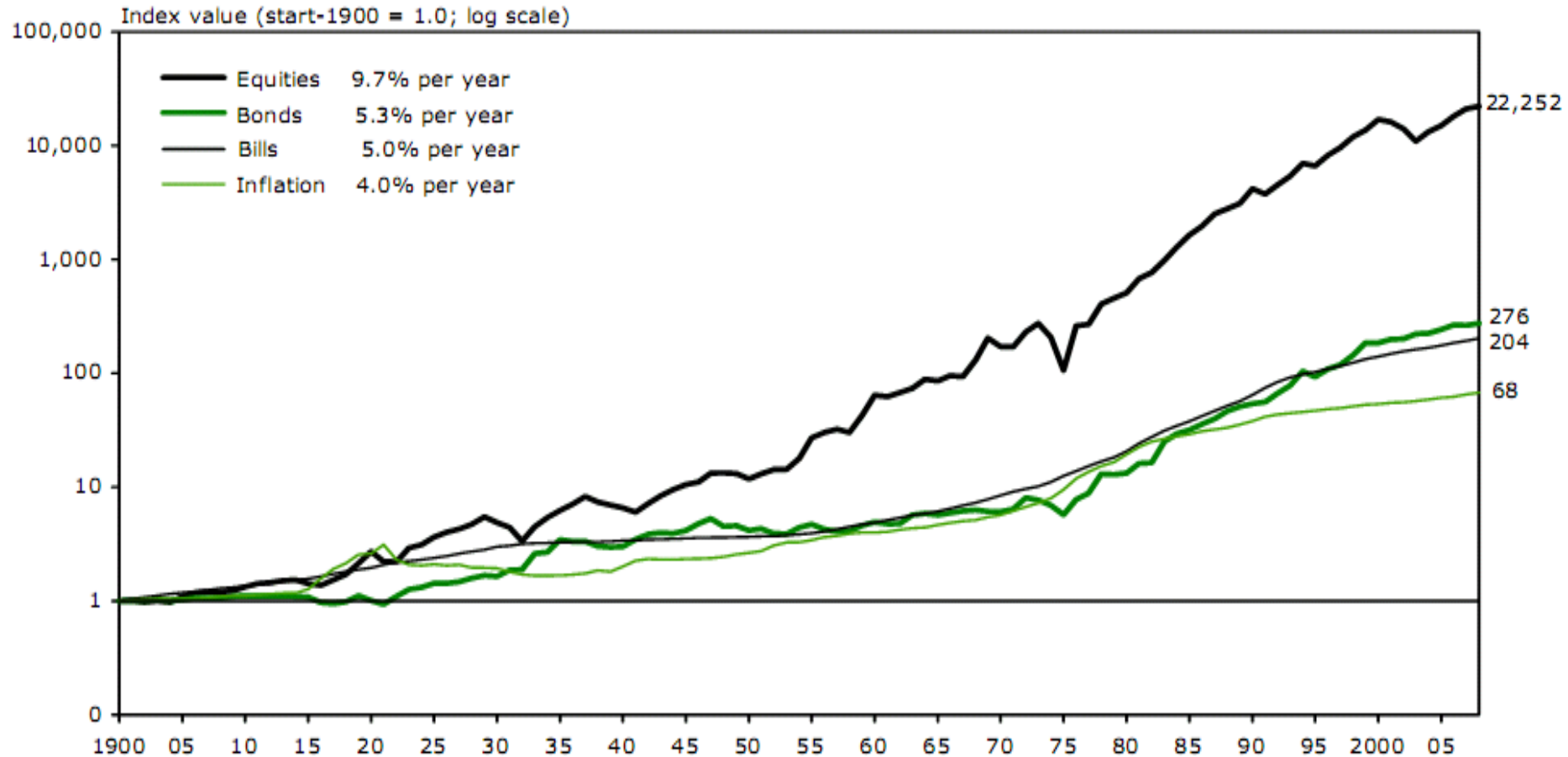
Dow Jones Industrial Average: 30 U.S. stocks (corresponding to 30 leading companies), price weighted

S&P500: 500 U.S. stocks, capitalization weighted

Stoxx 600: 600 european stocks, capitalization weighted

Stocks, bonds, bills and inflation in the UK from 1900 to 2007

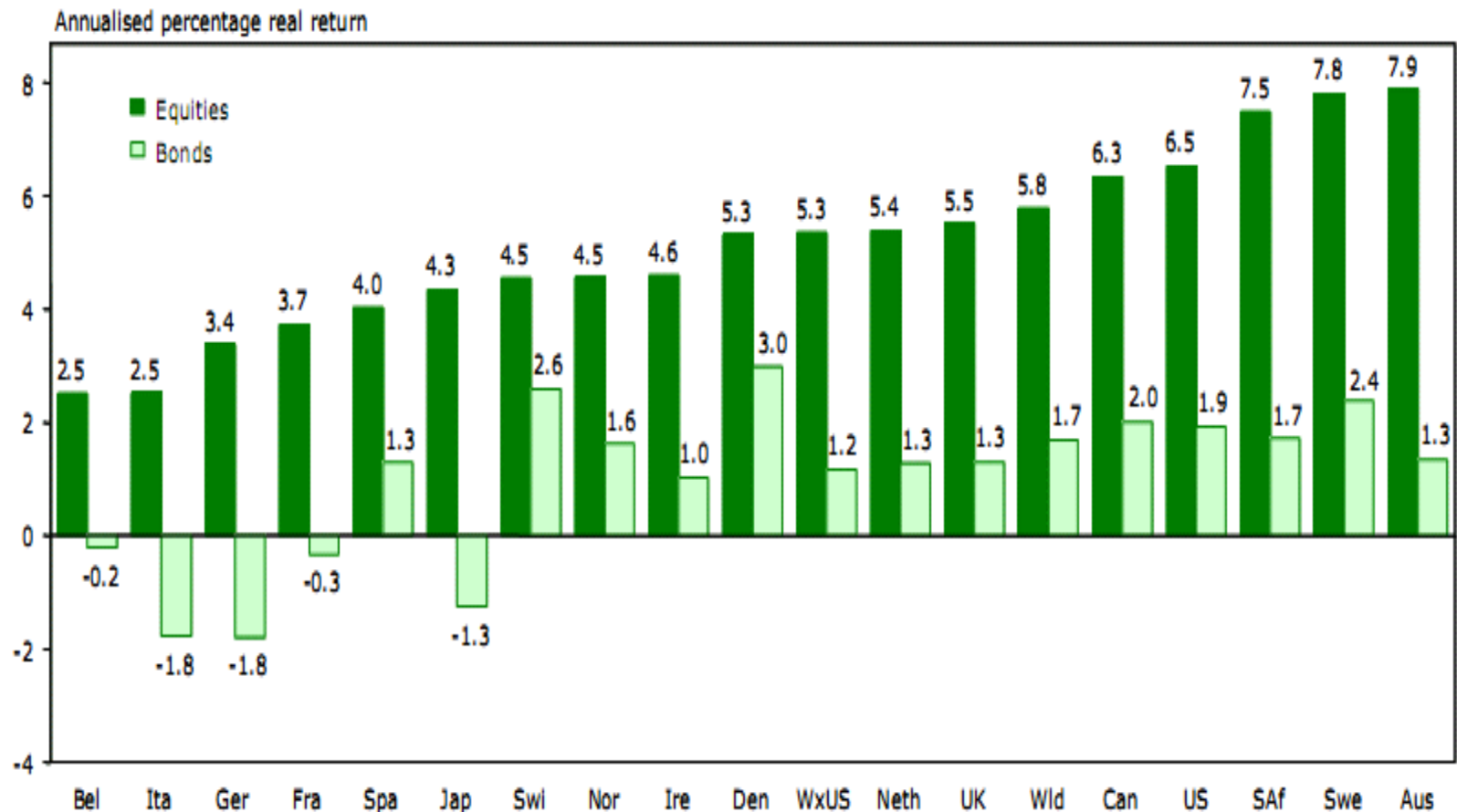
Figure 4: Cumulative returns on UK asset classes in nominal terms, 1900–2007



Source: ABN AMRO/LBS Global Investment Returns Yearbook 2008, chart 12

Annualized real (after inflation) returns of bonds and stocks: 1900-2007

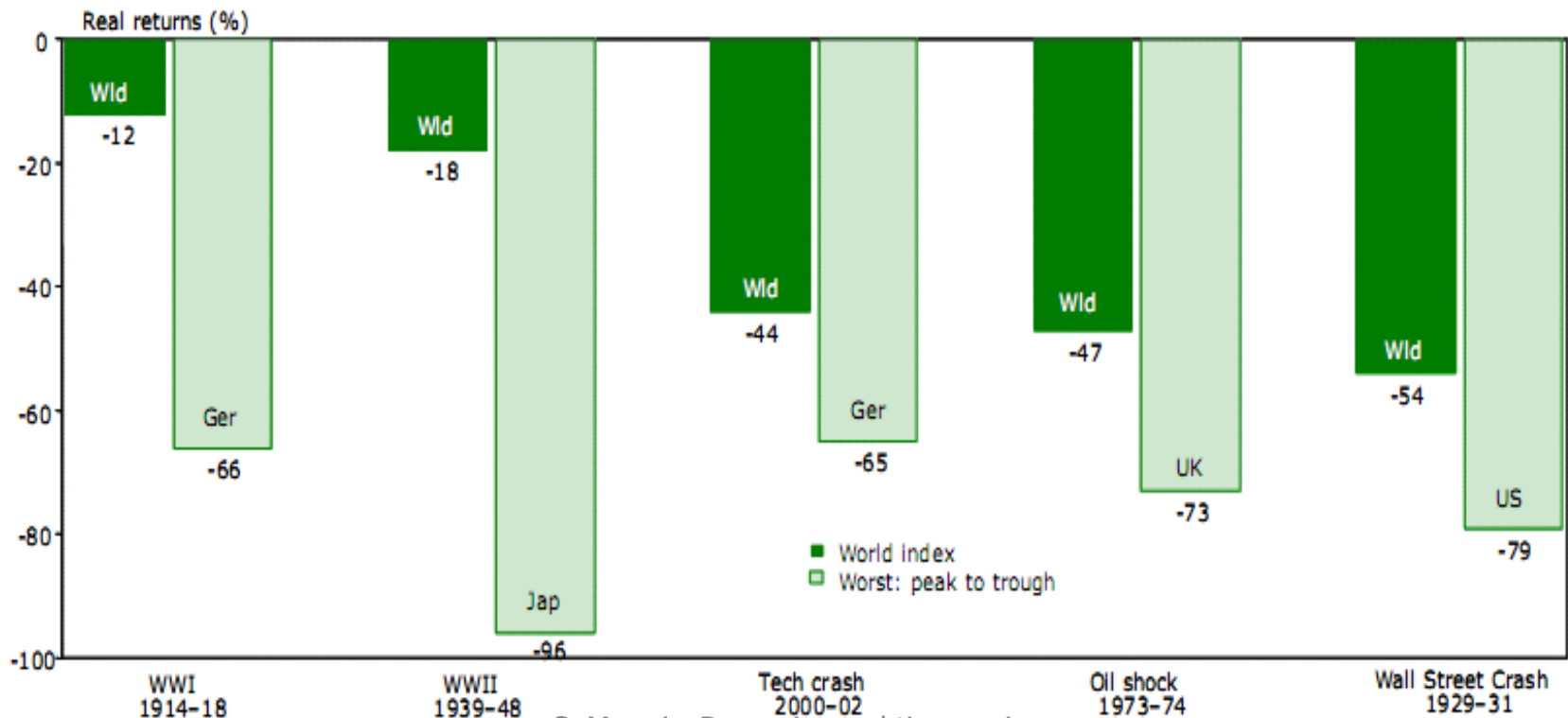
Figure 5: Real returns on equities versus bonds internationally, 1900–2007



Stock market crashes (before 2008)

GLOBAL INVESTMENT RETURNS BOOK 2008

Figure 6: Extremes of equity market history, 1900-2007



S. Marmi - Dynamics and time series -

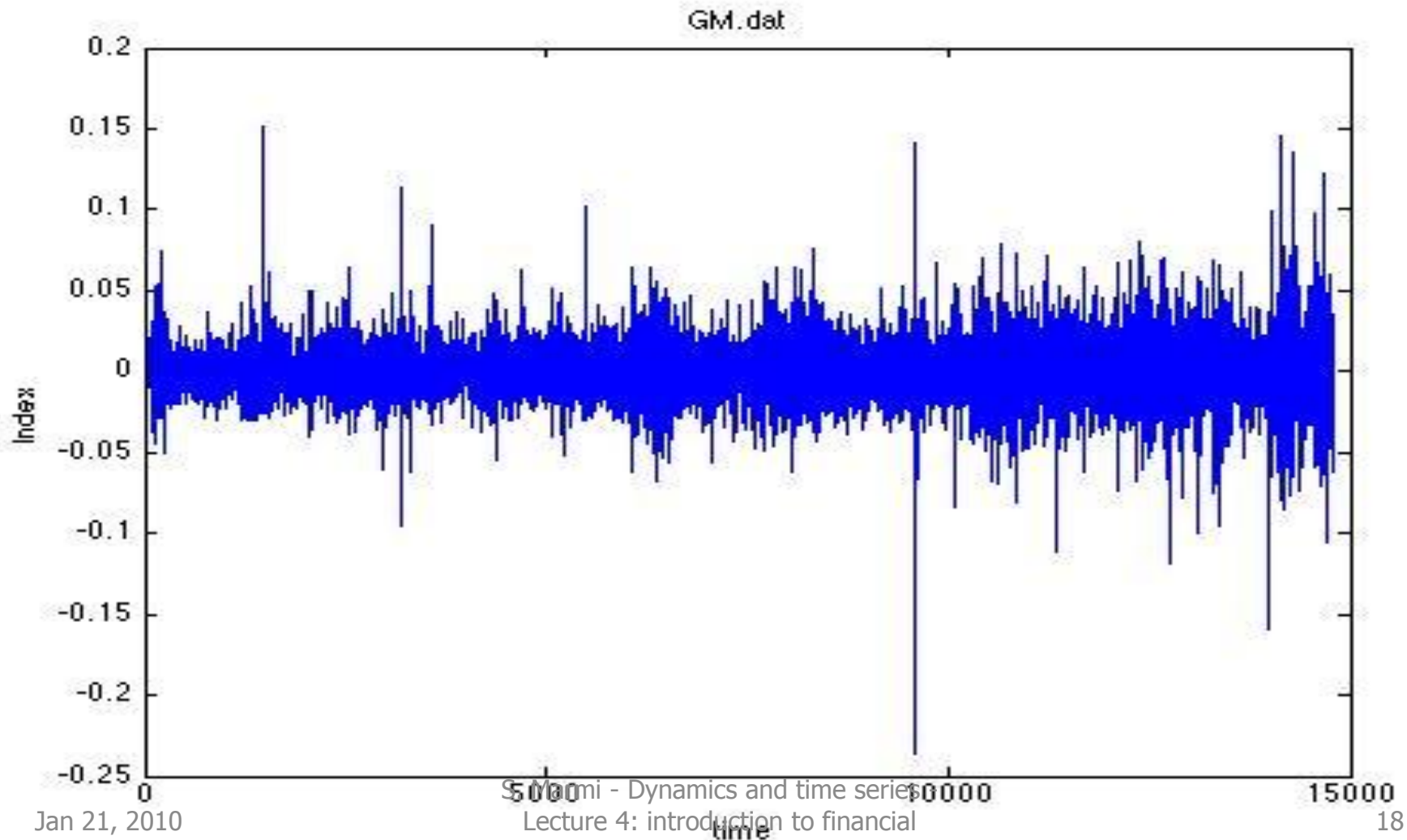
Volatility of stocks

During the period 1900-2007, UK's standard deviation of 19.8% places it alongside the US (20.0%) at the lower end of the risk spectrum. The highest volatility markets were Germany (32.3%), Japan (29.8%), and Italy (28.9%), reflecting the impact of wars and inflation.

Chicago Board Options Exchange Volatility Index, a popular measure of the implied volatility of S&P500 index options. A high value corresponds to a more volatile market and therefore more costly options, which can be used to defray risk from volatility. If investors see high risks of a change in prices, they require a greater premium to insure against such a change by selling options. Often referred to as the *fear index*, it represents one measure of the market's expectation of [volatility](#) over the next 30 day period.



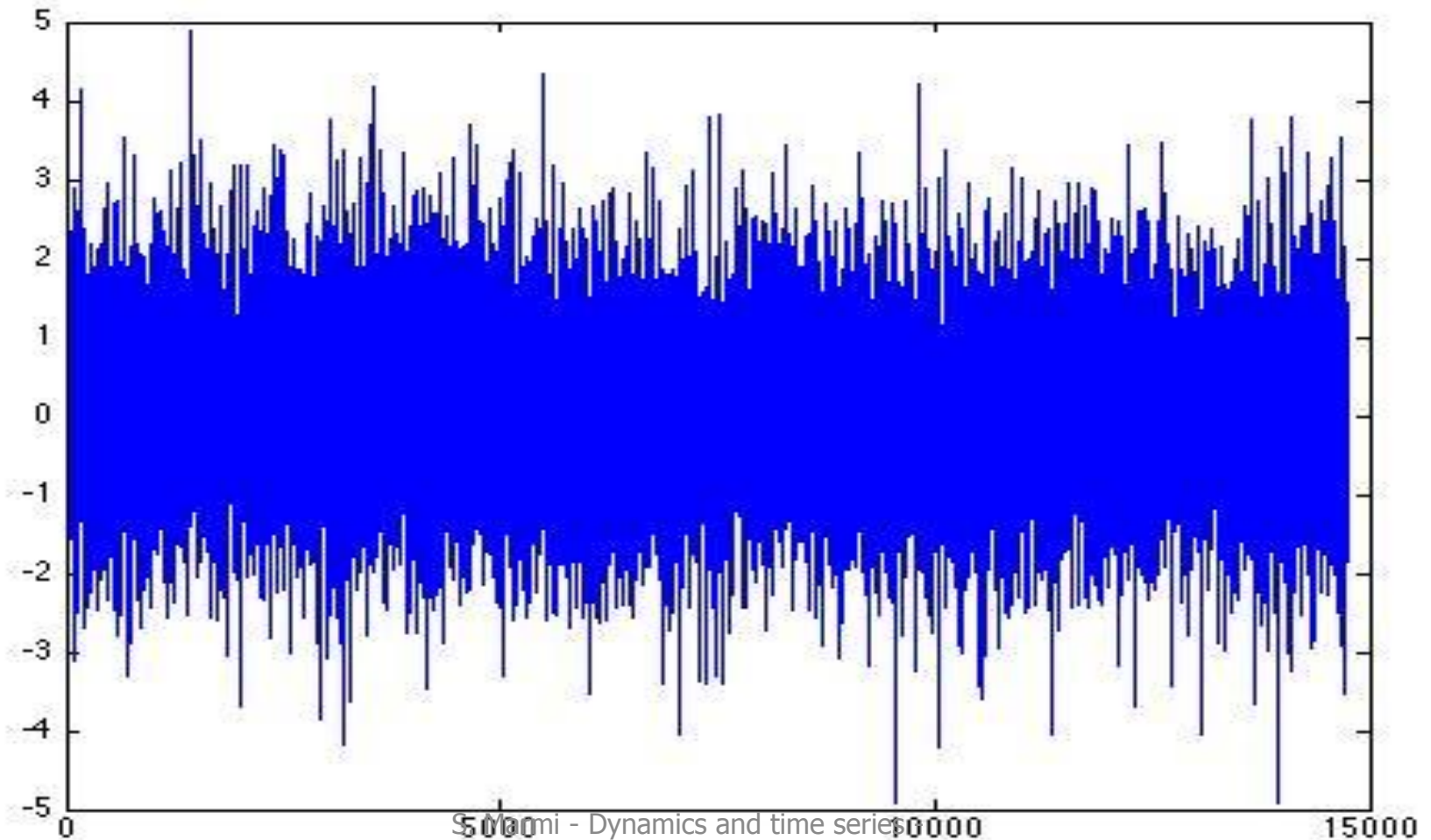
Daily returns of General Motors (1950-2008)



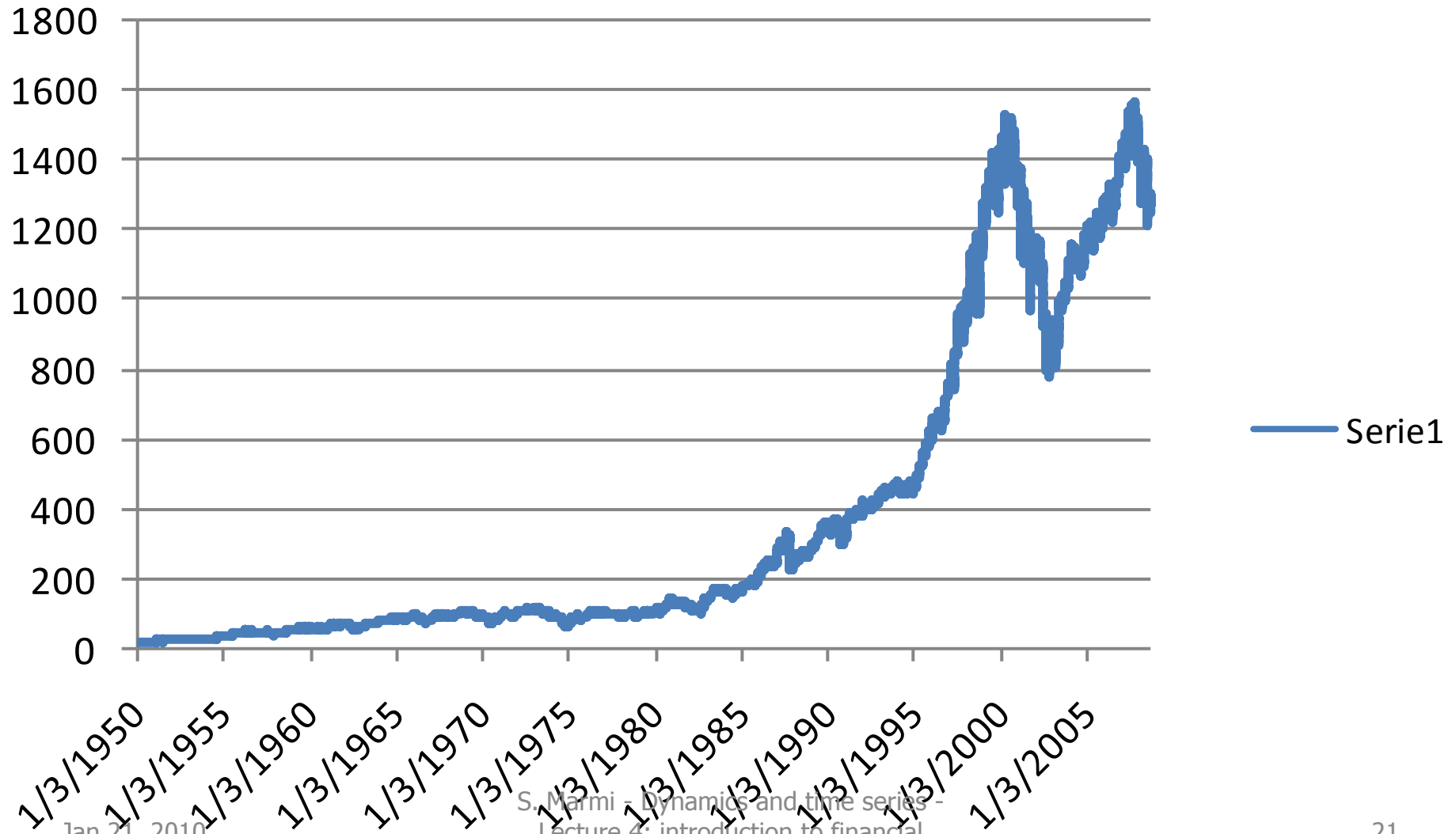
Volatility clustering

Time series plots of returns display an important feature that is usually called volatility clustering. This empirical phenomenon was first observed by Mandelbrot (1963), who said of prices that “large changes tend to be followed by large changes—of either sign—and small changes tend to be followed by small changes.” Volatility clustering describes the general tendency for markets to have some periods of high volatility and other periods of low volatility. High volatility produces more dispersion in returns than low volatility, so that returns are more spread out when volatility is higher. A high volatility cluster will contain several large positive returns and several large negative returns, but there will be few, if any, large returns in a low volatility cluster.

Daily returns of GM after normalization by short-term (25 days) volatility

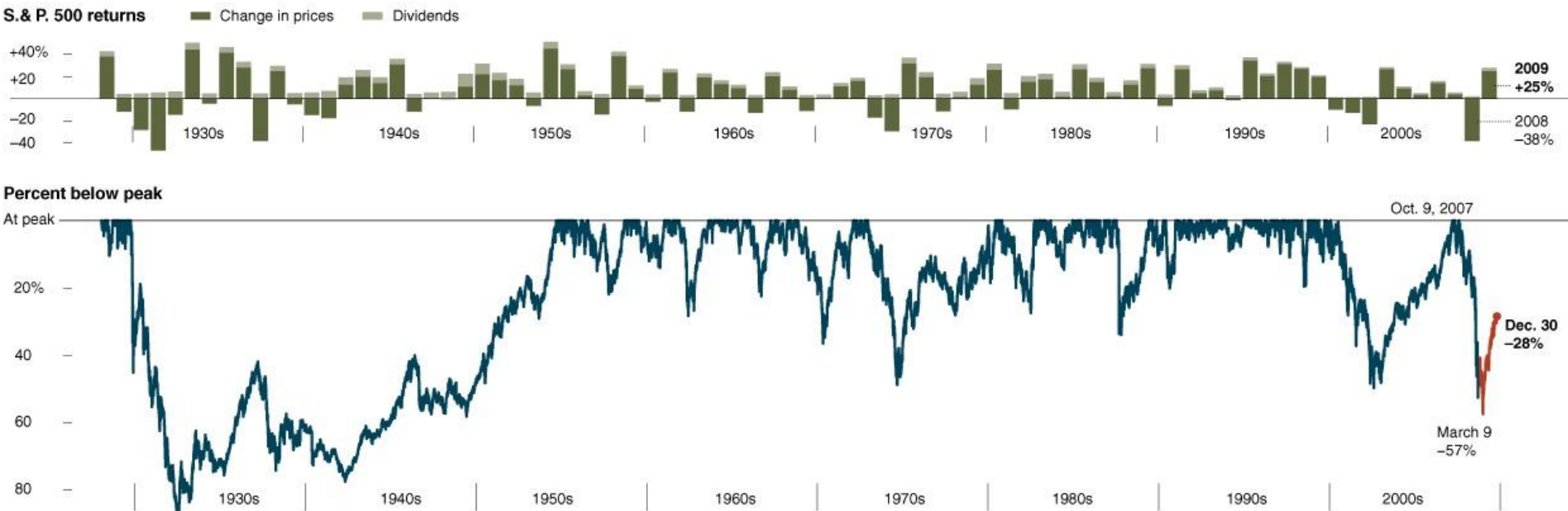


S&P500 1950-early 2008



The Difference a Year Makes

In March, the stock market was on track to have its worst year ever. Gains since then turned 2009 into one of the best years this decade, though one of the broadest measures of the market, the Standard & Poor's 500-stock index, still remains 28 percent below its October 2007 peak.



Source: Bloomberg

<http://www.nytimes.com/2009/12/31/business/31stox.html?th&emc=th>
<http://alfaobeta.blogspot.com>

Speculation and hedging

Speculators are investors who deliberately assume the risk of a loss, in return for the uncertain possibility of a reward. They bet on future events. For example, they will buy a stock because they think it will go up.

Hedgers are investors who trade so as to reduce their exposure to risk. For example, they will both buy and short a stock simultaneously.

The economic benefit of speculation

The well known speculator [Victor Niederhoffer](#), describes the benefits of speculation: “Let's consider some of the principles that explain the causes of shortages and surpluses and the role of speculators. When a harvest is too small to satisfy consumption at its normal rate, speculators come in, hoping to profit from the scarcity by buying. Their purchases raise the price, thereby checking consumption so that the smaller supply will last longer. Producers encouraged by the high price further lessen the shortage by growing or importing to reduce the shortage. On the other side, when the price is higher than the speculators think the facts warrant, they sell. This reduces prices, encouraging consumption and exports and helping to reduce the surplus.”

Another service provided by speculators to a market is that by risking their own capital in the hope of profit, they add liquidity to the market and make it easier for others to offset risk, including those who may be classified as hedgers and arbitrageurs.

Arbitrage

Arbitrage is a trading strategy which takes advantage of two or more securities being inconsistently priced relative to each other. In financial and economics theory arbitrage is the practice of taking advantage of a price differential between two or more markets or assets: striking a combination of matching deals that capitalize upon the imbalance, the profit being the difference between the prices. When used by academics, an arbitrage is a transaction that involves no negative cash flow at any probabilistic or temporal state and a positive cash flow in at least one state; in simple terms, a **risk-free profit**.

Advanced arbitrage techniques involve sophisticated mathematical analysis and rapid trading.

More arbitrage and market efficiency

The classical joke on arbitrage and market efficiency: A finance professor and a normal person go on a walk and the normal person sees a 100\$ bill lying on the street. When the normal person wants to pick it up, the finance professor says:

‘Don’t try to do that! It is absolutely impossible that there is a 100\$ bill lying on the street. Indeed, if it were lying on the street, somebody else would already have picked it up before you’ (end of joke).

How about financial markets? There it is already much more reasonable to assume that there are no 100 bills lying around waiting to be picked up. We shall call such opportunities of picking up money that is ‘lying around’ arbitrage possibilities. Let us illustrate this with an easy example.

Stock Returns

Let p_t be a representative price for a stock in period t (final transaction price or final quotation during the period). Assume that the buyer pays the seller immediately for stock bought .

Let d_t be the present value of dividends, per share, distributed to those people who own stock during period t . On almost all days there are no dividend payments $\rightarrow d_t = 0$. Sometimes dividend payments are simply ignored, so then $d_t = 0$ for all days t .

Three price change quantities appear in empirical research:

$$r_t^* = p_t + d_t - p_{t-1}$$

$$r'_t = (p_t + d_t - p_{t-1}) / p_{t-1}, \quad \text{simple net return (arithmetic)}$$

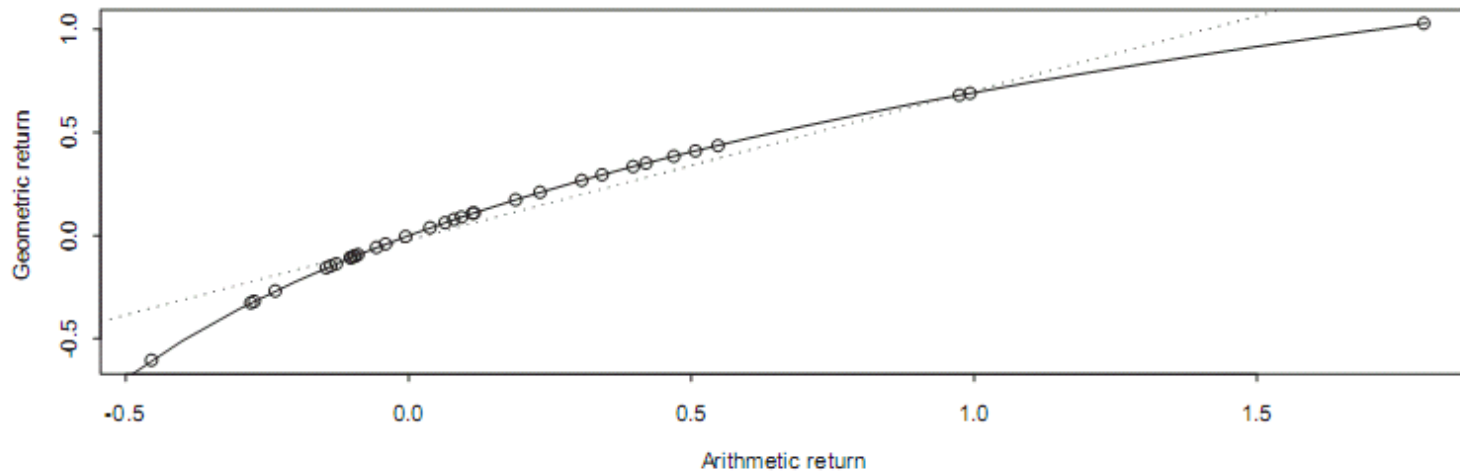
$$r_t = \log(p_t + d_t) - \log p_{t-1}. \quad \text{log returns (geometric)}$$

The return measures r_t and r'_t are very similar numbers, since

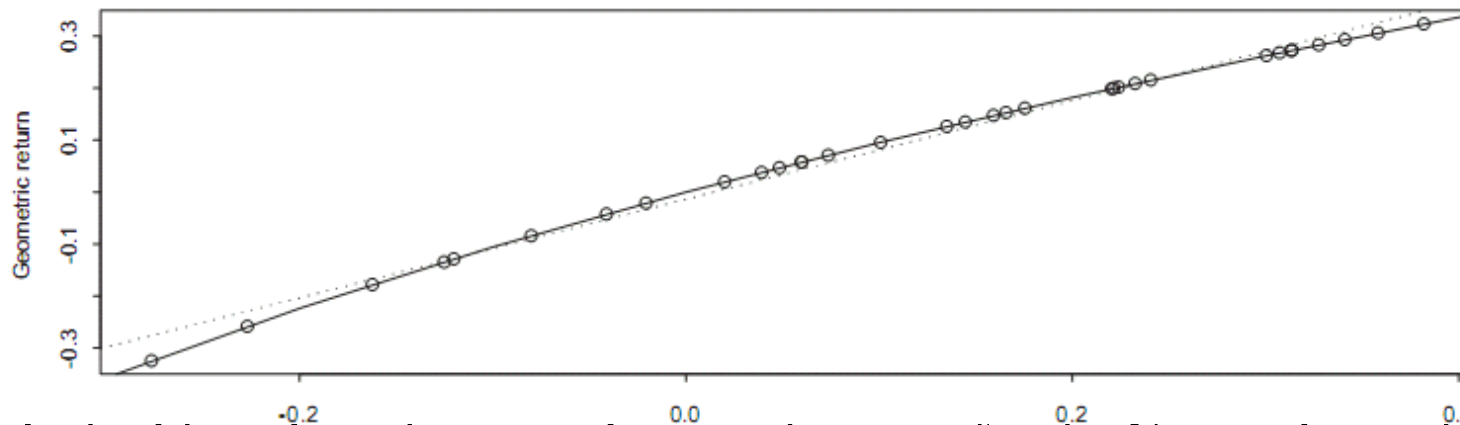
$$1 + r'_t = \exp(r_t) = 1 + r_t + \frac{1}{2} r_t^2 + \dots$$

and very rarely are daily returns outside the range from -10% to 10% . It is common to assume that single-period geometric returns follow a normal distribution.

Norway



USA



Historical arithmetic and geometric annual returns for the Norwegian and U.S. stock market (1970-2002). The historical annual volatilities in the two markets are very different: 18% for the U.S. market and 44% for the Norwegian market. From “Statistical modelling of financial time series: An introduction” K. Aas, X. Dimakos (2004) <http://www.nr.no/files/samba/bff/SAMBA0804.pdf>

Multiperiod returns

The multiperiod log return is simply the sum of the log returns.

Multiplying simple net returns then gives the return over a longer period (we ignore dividends for simplicity):

$$1 + r'_t[k] = p_t / p_{t-k} = \prod_{j=0}^{k-1} p_{t-j} / p_{t-j-1} = \prod_{j=0}^{k-1} (1 + r'_{t-j})$$

Over k periods the growth rate of the asset is the geometric mean of the returns

$$R[t,k] = \left(\prod_{j=0}^{k-1} (1 + r'_{t-j}) \right)^{1/k} - 1$$