

# Truthlikeness and graded similarity: some logic-based approaches

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The similarity-based notion of truthlikeness was proposed independently by Hilpinen (1976) within possible worlds semantics and by Tichý (1974) within propositional logic. The basic idea of this *similarity approach* is that the degree of truthlikeness of a sentence  $\varphi$  depends on the similarity between the states of affairs that are compatible with  $\varphi$  and the true state of the world. According to Niiniluoto (1987), we can consider the *truthlike* value of a sentence as its degree of “proximity to the truth”, even though it may be false. This degree should be given by the “distance” that separates (or dually, by the similarity between) the models of this sentence and the models of the “reality”.

Thus, this notion of truthlikeness can be regarded as a special case of the more general concept of similarity and its logical counterparts to some form of similarity-based reasoning, this last concept being often associated with reasoning by analogy which is an important form of non-demonstrative inference. Similarity-based reasoning aims at studying which kinds of logical consequence relations make sense when taking into account that some propositions may be closer to be true than others. A typical kind of inference which is in the scope of similarity-based reasoning responds to the form “if  $\varphi$  is true then  $\psi$  is *close* to be true”. The fact of  $\psi$  being close to (or approximately) true has nothing to do with a problem of missing information (uncertainty) preventing us to know whether  $\psi$  is true or false.

In the literature one can find qualitative (or comparative) and quantitative approaches to similarity-based reasoning. Comparative approaches aim at formalizing e. g. expressions like  $p$  is closer to  $q$  than  $r$ , like those of Nicod (1970), Lewis (1973), Williamson (1988) and Konikowska (1997). Quantitative approaches, that are based somehow on a numerical definition of degree

of truthlikeness or similarity following Niiniluoto (1987) and Weston (1987). This kind of approach, although not always within a formal logical framework, has been further developed by making use of Zadeh's fuzzy similarity relations as graded modelings of similarity relations, originally to be used in techniques of categorization and clustering. A key contribution in this direction was made by Ruspini (1991) who pushed forward the idea of similarity as one of possible semantics for fuzzy sets, where membership degrees are understood as similarity degrees to some of the prototypes of the given fuzzy set (Dubois and Prade, 1994). In this direction one can establish interesting links between similarity-based reasoning and fuzziness, see the references.

The aim of this talk is to survey a class of logical formalizations of similarity-based reasoning models where similarity is understood as a graded notion of truthlikeness. We basically focus on semantically-oriented approaches based on several notions of approximate entailments, providing different formalisations in terms of suitable systems of modal and conditional logics, including for each class a system of graded operators with classical semantics, as well as a system with many-valued operators. Finally, we also explore some nonmonotonic issues of similarity-based reasoning. Here are the main references the talk will be based on:

- [1] D. Dubois, F. Esteva, P. Garcia, L. Godo, and H. Prade. A logical approach to interpolation based on similarity relations. *International Journal of Approximate Reasoning*, 17:1–36, 1997.
- [2] F. Esteva, P. Garcia, L. Godo, and R. Rodríguez. A modal account of similarity-based reasoning. *International Journal of Approximate Reasoning*, 16(3-4):235–261, 1997.
- [3] L. Godo and R. Rodríguez. Graded similarity-based semantics for nonmonotonic inferences. *Annals of Mathematics and Artificial Intelligence* 34, 89-105, 2002.
- [4] L. Godo and R. Rodríguez. Logical approaches to fuzzy similarity-based reasoning: an overview. In *Preferences and Similarities*, G. Della Riccia, D. Dubois, R. Kruse, H.-J. Lenz (eds.). CISM Courses and Lectures, no. 504: Springer, pp. 75-128, 2008.